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THE MATHEMATICS TEACHER

Volume XXVIII Number 3

Edited by William David Reeve

Report of the Commission on Examinations in Mathematics to the College Entrance Examination Board

THE "Report of the Commission on Examinations in Mathematics" to the College Entrance Examination Board appearing on pages 154–166 of this issue of *The Mathematics Teacher* will be of particular interest to all teachers of secondary school mathematics.

While the College Entrance Examination Board syllabi in mathematics have been quite liberal in comparison with many courses of study throughout the country, certain teachers have complained of the hampering influences of extra mural examinations.

A careful study of this latest Report will show that the Commission has honestly endeavored to embody in its report the liberal tendencies in the teaching and testing of mathematics in the secondary school and, has done its best to remove the criticism referred to above.

The report of this Commission will be submitted to the *Committee of Review* at a meeting to be held April 9, 1935. The Committee of Review will welcome suggestions and criticisms from those interested. Such suggestions or criticisms should be sent to the College Entrance Examination Board, 431 West 117th Street, New York, N. Y.

The Mathematics Teacher is glad to cooperate in giving publicity to this forward looking report and congratulates the Commission on its recommendations

W. D. REEVE

Sales of Books on the Teaching of Arithmetic

By R. L. MORTON
Ohio University

ON SEVERAL occasions in recent years the writer has heard the remark that if the sales of a book designed for the professional preparation of teachers reached or exceeded five thousand copies within five years of the date of publication, the book was considered a success. Judged by this standard, books on the teaching of arithmetic are rarely unsuccessful.

The writer has undertaken to collect and compile statistics on the sales of books dealing with the teaching of arithemtic. Information was collected through correspondence with authors and publishers, usually the publishers with the knowledge and consent of the authors. Detailed sales figures have been assembled for 22 titles representing the work of 21 authors and the publications of 13 companies.

The publications considered—The 22 titles are listed in Table I. With each is given the author or authors, the publisher, the year for which first sales figures were received (the approximate date of publication) and the number of years for which sales data were collected.

Of the 13 publishing companies represented, nine make up their reports for the year ended December 31. Two consider the year as ending April 30, one July 1, and one July 31. In order that the sales figures for these publications might be combined by years, data supplied by the four companies whose fiscal years do not end December 31 were assigned to the year which they most nearly fitted. Data for the year ended April 30, 1929, for example, were considered to be 1928 data.

Publishers do not ordinarily release information relative to the sales of their publications. The writer assured each co-operating publisher and author that the data would be treated confidentially so far as the sales of individual books were concerned. For this reason, totals, averages, and trends only are reported in this article. Not all publishers approached would supply the requested data. Those acquainted with the literature of this field will probably agree, however, that the sales figures for the 22 titles listed in

Table I furnish a rather complete picture of professional interest in this field in recent years.

TABLE I

Titles of Books on the Teaching of Arithmetic for Which Sales Data Were Received, with Authors, Publishers, and Years

Author or Authors	*Title*	*Publisher*	*Year of First Figures*	*Number of Years*
Brown-Coffman	How To Teach Arithmetic	Row, Peterson	1915	10
Brown-Coffman	The Teaching of Arithmetic	Row, Peterson	1925	8
Buckingham-Osburn	Searchlight Arithmetics Introductory Book	Ginn	1927	6
Clark-Otis-Hatton	First Steps in Teaching Number	World Book	1929	4
Klapper	The Teaching of Arithmetic	Appleton	1917	16
Lindquist	Modern Arithmetic Methods and Problems	Scott, Foresman	1917	16
Losk-Weeks	Primary Number Projects	Houghton Mifflin	1923	10
McNair	Methods of Teaching Modern Day Arithmetic	Badger	1923	10
Morton	Teaching Arithmetic in the Primary Grades	Silver Burdett	1927	6
Morton	Teaching Arithmetic in the Intermediate Grades	Silver Burdett	1927	6
Newcomb	Modern Methods of Teaching Arithmetic	Houghton-Mifflin	1926	7
Osburn	Corrective Arithmetic	Houghton-Mifflin	1927	6
Osburn	Corrective Arithmetic II	Houghton-Mifflin	1929	4
Overman	Principles and Methods of Teaching Arithmetic	Lyons-Carnahan	1920	13
Overman	Arithmetic for Teachers and Teacher-Training Classes	Lyons-Carnahan	1923	10
Roantree-Taylor	An Arithmetic for Teachers	Macmillan	1925	8
Stone	The Teaching of Arithmetic	Sanborn	1918	15
Stone	How to Teach Primary Number	Sanborn	1922	11
Suzzalo	The Teaching of Primary Arithmetic	Houghton-Mifflin	1912	21
Taylor	Arithmetic for Teacher-Training Classes	Henry Holt	1927	6
Thorndike	The New Methods in Arithmetic	Rand McNally	1922	11
Thorndike	The Psychology of Arithmetic	Macmillan	1921	12

Sales data were secured for each publication by years from the date of publication, except that in the case of one title the data

were supplied for two-year periods. In this latter case, the writer made an estimate of the number of copies sold in each of the two years covered by one report time, the estimate being made after a study of the sales trends of this title and of other titles. Sales figures for the earlier of the Brown-Coffman books were given for the years 1915–1924 only, the second Brown-Coffman book apparently being considered as a revision of the first.

With the minor exceptions noted, sales figures were secured for each title from the year of publication to and including the year 1932. The total sales of these 22 titles for the periods covered by the reports were 521,217 copies.

Average sales by years—Table II shows the average sales by years for 22 or fewer titles for the first 10 years after publication. The average sales figures in Table II are not averages for calendar

TABLE II

Total and Average Sales Figures by Years for Twenty-Two or Fewer Titles

Year	*Total Sales*	*Number of Titles*	*Average Sales*
1	68,118	22	3,096
2	107,914	22	4,905
3	83,995	22	3,818
4	67,118	22	3,051
5	56,394	20	2,820
6	44,551	20	2,228
7	27,021	15	1,801
8	21,228	14	1,516
9	14,774	12	1,231
10	12,528	12	1,044

years but averages for publication years. The total first year sales for these 22 titles were 68,118 copies. These were sold in calendar years ranging from 1912 to 1929 as is shown in the fourth column of Table I.

Apparently, the second year is the best year in the life of a publication in this field. It should be remembered, however, that the figures given for the first year represent figures for less than a full year for most of these 22 titles and that the increase in popularity from the first year to the second is less real than apparent. After the second year a very definite decline is seen. The average number of copies sold drops from nearly five thousand to about one thousand in the tenth year.

Table II shows average sales by years. Naturally, no one publication has had just this average experience. Indeed, the publications whose sales figures were studied have had markedly varying experiences. Not only have some been much better received than

have others but also some have maintained the popularity which they have been able to win much more successfully than have others.

Figure I shows graphically the sales figures of two publications for the first ten years after publication. The solid line in this figure represents the experience of a book which was quite well received when it appeared but which suffered a rapid decline in sales until by the end of the tenth year it had almost disappeared from the

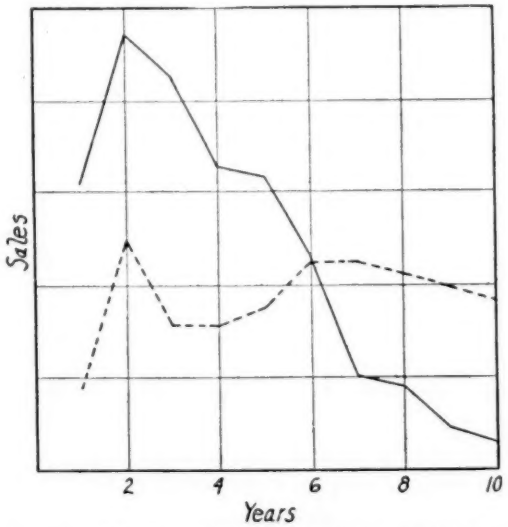

Fig. I. Two Publications with Markedly Varying Sales Records.

market. The other book, represented by the broken line, maintained the position which it was able to win with the public remarkably well until the end of the ten-year period. Both lines show a sharp increase in sales in the second year and a falling off in the third year. After the third year, however, the course of the two lines is very different. The sales records for the first three years suggest that one of these books is much more successful than is the other. However, it is probably safe to predict from the ten-year record here portrayed that eventually the book with the less promising beginning will show a better total sales record than will its competitor.

Effect of the depression—One purpose of the investigation was to show the effect of the depression upon the sales of books in this field. On this point, the data of Table III are illuminating.

TABLE III

Total Sales for the Ten-Year Period, 1923 to 1932

Year	*Number of Titles*	*Total Sales*	*Per Cent Change*
1923	12	38,400	—
1924	13	40,775	+6.2
1925	13	41,168	+1.0
1926	14	42,142	+2.4
1927	19	57,019	+35.3
1928	19	48,864	−14.3
1929	21	39,297	−19.6
1930	21	38,811	−1.2
1931	21	28,257	−27.2
1932	21	17,978	−36.4

This Table shows total sales for the ten years, 1923–1932. In 1923, the 12 titles then before the public showed a total sales record of 38,400 copies. The number increased slowly in 1924, 1925, and 1926 but we may think of a typical year's record at that time as about forty thousand copies.

It will be seen that there was a marked increase in sales in 1927. This was the banner year of all the years here under review. This condition may perhaps be explained in part by the fact that we were living in prosperous times then and in part, perhaps, by the fact that states and communities were gradually increasing the training requirement imposed upon teachers and prospective teachers. But the chief reason for the 1927 increase is probably found in the fact that five new titles appeared in 1927. When new books are adopted by teacher-training institutions, relatively large orders go to the publishers and there is temporarily a lack of second-hand books. New books find their way into state reading circle lists. They are purchased by libraries, by boards of education, and by individuals. In no other year of the period covered by this report did so many new books appear as in 1927.

Table III indicates that the total sales figure for 1928 was 48,864 copies, a decline of 14.3 per cent from the record of the preceding year. In 1929, there was a decline of 19.6 per cent from the 1928 figure. The figure for 1929 is 31.1 per cent less than that for 1927. By 1929, total sales were again approximately forty thousand copies.

Apparently the effect of the depression upon sales of books in this field was not very marked until 1931 when there was a decline of 27.2 per cent from the sales record of the preceding year. The declines in 1928 and 1929 can hardly be called depression declines since the first evidences of the beginning of the depression were observed in the summer of 1929. The 1928 and 1929 declines are

probably due to the natural reaction following the introduction of five new titles in 1927. In other words, it is hardly probable that the business represented here was seriously affected by the depression until late in 1930.

It cannot be denied that the 1931 decline, a decline which brought the total sales figure down to a level far lower than had been experienced for many years, was serious. Furthermore, few would deny that the depression was the major cause of this loss. The total sales figure in 1931 is 29.4 per cent less than forty thousand, the number which has been chosen as typical for several years, and 50.4 per cent less than the record sales total for 1927.

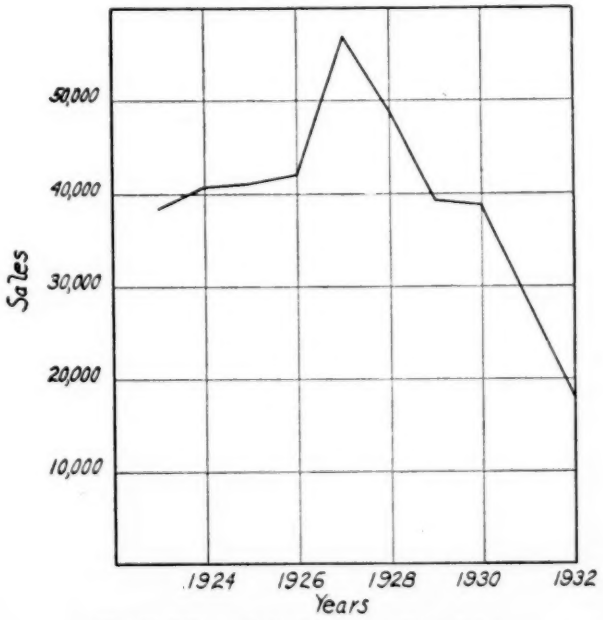

Fig. II. Effects of the Depression Upon Sales.

An even sadder state of affairs is revealed for 1932. The total sales for 21 titles in this year amounted to only 17,978 copies. This is a decline of 36.4 per cent from the poor record for 1931, as is shown in Table III. The 1932 number is only 44.9 per cent of 40,000, the number already suggested as typical of a normal year's sales. In other words, the 1932 record is 55.1 less than 40,000. It is 68.5 per cent less than the 1927 record of 57,019 copies.

The data of Table III are shown graphically in Figure II. The

fact that the line shown in Figure II is directly sharply downward in 1932 does not indicate that the downward trend will continue at the same rate. The writer predicts that the 1933 total will be less than that of 1932 but that the slope of the line in Figure II extended will be considerably reduced.

Summary.—Sales data for 22 books dealing with the teaching of arithmetic, representing the publications of 13 companies, reveal the following facts:

1. The average book is sold to the extent of about three thousand copies in its first year.

2. The second year is the best year in the life of a publication in this field. Average sales in this year amount to nearly five thousand copies.

3. After the second year, there is, on the average, a gradual decline in the number of copies sold per year. The average number sold is about three thousand in the fourth year, two thousand in the sixth or seventh year and one thousand in the tenth year.

4. Individual publications vary greatly, not only in the number of copies sold but also in the extent to which they last. Whereas, on the average, the number of copies sold decreases each year after the second, some publications rally after the third or fourth year and experience an increase in sales.

5. Total sales for 22 titles through 1932 amounted to a little over a half-million copies.

6. In a typical year prior to the time when the influence of the depression was felt, the total sales of the books whose records were studied were about forty thousand copies.

7. The banner year for sales of books in this field was the year 1927. In this year, the total sales for 19 titles were over fifty-seven thousand copies.

8. Sales of books in this field have experienced a marked decrease in recent years, due to the depression. The effects of the depression were not greatly felt until 1931 but the total sales for 1932 were less than half of those for the typical pre-depression year and less than a third of those of the banner year, 1927.

9. "On the average," a book dealing with the teaching of arithmetic is sold to the extent of about twenty-five thousand copies in the first ten years.

10. It is estimated that the downward trend in sales experienced in 1931 and 1932 has continued in 1933 but at a reduced rate.

The Need for Reorientation of Mathematics in the Secondary School

By P. W. HUTSON
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ANY GENERAL shift in educational theory necessitates a general readjustment of educational practice. That is axiomatic. If our canvass of the need for reorientation of mathematics suggests the abandonment of long-cherished conceptions of value or method and the acceptance of unfamiliar goals and means of attaining them there may be some comfort in the realization that the specialists in other fields of learning are facing similar problems of adjustment. It is my conviction that all the subject fields of secondary education can be considered from the angle of the topic of this afternoon's discussion quite as profitably as mathematics.

Perhaps there is no better point from which to start a brief exposition of modern educational theory than from the following words penned by John Dewey thirty-five years ago.[1]

> It [the old education] may be summed up by stating that the center of gravity is outside the child. It is in the teacher, the textbook, anywhere and everywhere you please except in the immediate instincts and activities of the child himself. . . . Now the change which is coming into our education is the shifting of the center of gravity. It is a change, a revolution, not unlike that introduced by Copernicus when the astronomical center shifted from the earth to the sun. In this case the child becomes the sun about which the appliances of education revolve; he is the center about which they are organized.

The elaboration of this point of view has brought us, since the turn of the century, to a host of concepts which redefine the educational task. For instance, with the pupil as the center of reference, the logical organization of subject matter must give way to a psychological organization. The sequence which the finished scholar is likely to think most economical is actually the most wasteful. Logical organization is something which grows out of much experience, but it cannot be taken as a starting point in learning. The starting point must be the learner's felt need, his

[1] *The School and Society*, p. 47. University of Chicago Press, 1899.

deep-lying interests. We must, accordingly, project ourselves into the pupil's world, arouse his curiosity concerning it, and guide him in experiences which are real and meaningful to him in exploring it. Such experiences may not only ignore the logical sequence of a subject; they are also quite likely to cut indiscriminately across the boundary lines of subjects. Where they will lead to is the question which dismays the subject matter specialist. But in Kilpatrick's assurance that proper respect for the pupil's purposes does not mean "a child's doing what he wishes," but "wishing what he does,"[2] there is hope that growth will truly eventuate. In other words, the psychological organization does not by any means dispense with the teacher's leadership. It simply implies a notably different conception of the teaching job.

Another implication of the acceptance of the pupil as the central point of reference in the educative process is embodied in the expression that education is for present living. The older theory was that education is for adult living, that childhood is a period of preparation for carrying on the activities of adulthood. In pursuance of that theory we attempted to teach pupils things that we adults know, but for which they, in their state of immaturity, have no intellectual use. Such objectives we have come to call deferred values, and to contend that actually they cannot be realized. Modern theory does not deny that education is preparation for adulthood but says that the best assurance of attaining that goal is attention to the needs of present growth. Whether the pupil is 12 or 15 or 18 years of age, education will help him to live as competently and as richly as he is capable of living at his present age.

Going a step further, modern theory insists that education is not properly thought of as *preparation* for life; it is thought of as life itself. Preparation for college is an aim which has long dominated the secondary school. Certain subjects have been and still are held to be particularly efficacious in performing this function. Recent investigations,[3] however, have made it clear that the type of curriculum pursued in high school has little influence on college success. The colleges are recognizing this fact and are gradually shifting their entrance requirements to measurements of ability

[2] W. H. Kilpatrick, *Foundations of Method*, p. 207. Macmillan Co., 1925.

[3] For example, *The Type of High-School Curriculum Which Gives the Best Preparation for College*, Bulletin of the Bureau of School Service, Vol. II, September, 1929. Lexington, Kentucky: University of Kentucky.

and purpose. Such release for the high schools permits their larger application of the concept that education should progressively utilize present possibilities of the pupil.

A third consequence of the shift of the center of gravity from the subject to the child is that it now becomes possible and necessary to utilize only such *parts* of a subject as are at any time essential to the child's growth. Formerly, the subject specialists constructed the curriculum; they built each course so that it presented in logical order the fundamental facts and principles of the subject. The idea was that the learners, despite their varying needs and purposes, would master the subject and thereafter draw upon it in accordance with their needs. The modern theory has called for analysis of the activities of individuals or groups to determine what elements of the subject should be taught those in training for such activities. The activity-analysis movement has profoundly affected the curriculum of the elementary school and will undoubtedly play an increasing rôle in the remodeling of the secondary-school curriculum, although the technique has its limitations which must be observed.

Finally, acknowledgment of the learner as the central consideration has contributed to the acceptance of the principle that individual differences should be respected. Psychology has defined individual differences as to character and extent. The increasing complexity of society is forcing the whole range of human differences into the secondary school. Modern educational theory demands that school opportunities and practices be such that each child will be developed according to his capacities and interests.

There is a second marked shift in viewpoint which we must grasp if we would see the barest elements of present-day educational theory. That shift began with the rejection of faculty psychology. As long as it was held that the mind was made up of relatively independent faculties, such as the faculties of memory, attention, reasoning, imagination, there seemed justification for the assumption that valuable general abilities might be developed through training in specific fields. What was meant, therefore, when it was said that geometry develops the reasoning power, was that through exercise of the mind on the specific material of geometry, one's capacity for reasoning on the choice of a career, a house, an automobile, or for reasoning on any of the manifold problems of life, would be improved. From such a view it was but a step to the

natural correlate that a study which brought about such a general improvement did not need to have specific or practical values. With the schools in the grip of such a theory, it is small wonder that Mr. Dooley could voice the educational philosophy that "It doesn't matter what you study as long as it's hard."

But developments in the field of psychology brought about the discard of faculty psychology and cast doubt on the whole conception that efficiency improved through exercise on material of one kind can be transferred to material in a different field. The arguments on this subject are too extended to be taken up here. It must suffice to say that the trend today is to require every subject to justify itself in terms of specific values for living. For reasoning in the various situations of life, one must have experience with those situations. We reason in a given situation on the basis of the experience we have had.

Some illumination of the claims of formal discipline has been contributed by Thorndike's experiment[4] to determine the influence of the various school studies on mental growth.

> The experiment . . . consisted of an examination in May, 1922, and a re-examination in May, 1923, of 8,564 pupils who, in May, 1922, were in Grades IX, X, and XI. The two examinations were alternative forms of a composite of tests of 'general intelligence' that are in common use, plus certain ones added in order to have measures with spatial as well as verbal and numerical content. . . . Each pupil who took both examinations recorded the subjects which he studied during the school year, September 22, 1922, to June 23, 1923; and the gains made were put into relation with the subjects studied. For example, we compare the gains for the pupils who studied English, history, geometry, and Latin during the year with the gains for the pupils who studied English, history, geometry, and shop-work.[5]

The results showed negligible differences between subjects with respect to their production of general mental growth. Thorndike's conclusion was that

> By any reasonable interpretation of the results, the intellectual values of studies should be determined largely by the special information, habits, interests, attitudes, and ideals which they demonstrably produce. The expectation of any large differences in general improvement of the mind from one study rather than another seems doomed to disappointment. The chief reason why good thinkers seem superficially to have been made such by having taken certain school studies, is that good thinkers have taken such studies, becoming better by the inherent tendency of the good to gain more than the poor from any study. When the good thinkers studied

[4] E. L. Thorndike, "Mental Discipline in High-School Studies," *Journal of Educational Psychology* (January, February, 1924), Vol. XV, pp. 1–22, 83–98.

[5] *Ibid.*, p. 1.

Greek and Latin, these studies seemed to make good thinking. Now that the good thinkers study Physics and Trigonometry, these seem to make good thinkers. If the abler pupils should all study Physical Education and Dramatic Art, these subjects would seem to make good thinkers. These were, indeed, a large fraction of the program of studies for the best thinkers the world has produced, the Athenian Greeks.[6]

Now we turn to the question of how mathematics has met the shift in educational theory. The authors of books on the teaching of mathematics which appeared 20 or 25 years ago—Smith, Schultze, and Young—all laid great claim to the general disciplinary values of mathematics study. They went further and largely denied direct or specific values in mathematics. The average citizen, they said, has but little need of any mathematical facts, or opportunity to use them, beyond the elements of arithmetic. Even for many specialists, such as navigators, surveyors, skilled mechanics, and engineers, the needed facts of algebra, geometry, and trigonometry can be taught in relatively few lessons. In the same period, Milliken said, "There is no mathematics needed in elementary physics even as it is now, except the simplest algebraic equations with one unknown, and the single geometrical proposition of the proportionality of the sides of similar triangles."[7]

The report of the National Committee on Mathematical Requirements,[8] published in 1923, signified a considerable acknowledgment of the shift in educational theory. In its recommendations that no mathematics be required of all pupils beyond the ninth grade, that the mathematics of the junior high school years should introduce pupils to the simpler elements of the several branches of secondary-school mathematics, and that junior high school mathematics should be organized with little reference to courses to be taken in succeeding years and with no specific consideration for college entrance requirements, the Committee took decided steps toward the practical fulfillment of current educational philosophy. The Committee advocated that "the problems [of the ninth grade] must be 'real' *to the pupil*, must connect with his ordinary thought, and must be within the world of his experience and interest. . . . There should be, moreover, a conscious effort to correlate

[6] *Ibid.*, p. 98.

[7] R. A. Milliken, "Elimination of Waste in the Teaching of High-School Science," *School and Society* (January 29, 1916) Vol. III, p. 167.

[8] *The Reorganization of Mathematics in Secondary Education*. The Mathematical Association of America, Inc., 1923.

the work in mathematics with the other courses of the curriculum, especially in connection with courses in science."[9]

With regard to the elective courses of the senior high school, the Committee urged that it is appropriate in those years to pay some attention to students' vocational and later educational needs. They advanced a considerable argument in the attempt to buttress the concept of deferred values. Apparently accepting the doctrine of specific or direct values, the Committee said: "The material for these years should include as far as possible those mathematical ideas and processes that have the most important applications in the modern world."[10] When a layman examines the outlines, however, he feels that "as far as possible" was not very far, that innovations were to be encouraged only to the degree that they did not interfere with subject matter in its logical organization. It seems likely that the Committee leaned primarily upon their statement of "disciplinary aims" to justify the senior high school cycle. The members of this audience will recall that the Committee listed the aims of mathematical instruction in three categories, —practical, disciplinary, and cultural. As I read the disciplinary aims, they mean quite decidedly the improvement of general ability in reflective thinking, and they state the development of such generalized traits as "an attitude of enquiry," "concentration and persistence," and "a love for precision, accuracy, thoroughness, and clearness." It is only fair to the Committee to say that Thorndike's experiment was not reported until after the publication of their statement.

The National Survey of Secondary Education, completed in 1932, included a monograph on mathematics.[11] The procedure followed in the survey was to locate outstanding schools and to study their offering in mathematics by visitation and analysis of their course outlines. The outlines for about four-fifths of the junior high schools showed selection and organization of materials with reference to children's present needs and interests, and about the same percentage reflected direct future life needs. Of the senior high school outlines only six per cent reflected regard for present interests, and twelve per cent, future needs. The investigator character-

[9] *Ibid.*, p. 28. [10] *Ibid.*, p. 33.

[11] Edwin S. Lide, *Instruction in Mathematics*. National Survey of Secondary Education Monograph No. 23. United States Office of Education Bulletin No. 17, 1932.

ized 47 per cent of the junior high school outlines as "psychological," as compared with 15 per cent of the senior high school outlines. Ninety per cent of the junior high school outlines were found to contain "some materials chosen from other subject fields," which suggests plainly the teaching of mathematics in natural social settings, while but twelve per cent of the senior high school outlines were so characterized.

That mathematics beyond the eighth grade can be adequately adapted to the abilities, needs, and interests of the present school population, or that it is necessary to the development of all pupils, apparently is now in doubt. Of the schools canvassed in the National Survey report, slightly more than half required mathematics in some form in the ninth grade. That the materials advocated for all pupils in the ninth grade by the National Committee on Mathematical Requirements in 1923 are not satisfactory for the present-day ninth-grade population is quite plainly suggested in a study just published in the December *School Review*.[12] I refer to Lindquist's report of the results of the comprehensive examination in elementary algebra administered in connection with the Iowa Every-Pupil Achievement Testing Program to 9,034 ninth-grade pupils just completing the course in 230 Iowa high schools. The test was based on the simpler elements of algebra, probably not going beyond the matters outlined for junior high school years by the Committee of 1923. It is not possible to present the results here, but the pupils' scores were very low, amply justifying Lindquist's conclusion "That a significant proportion of high-school pupils are, by reason of mental ability, previous training, and present motivation, incapable of deriving enough of value from ninth-grade algebra as it is now generally taught to justify its being required of all pupils."[13] The New York State Department of Education has recently taken action, in recognition of such a condition, to omit all requirement of mathematics for the State High School Diploma.

As a final step in preparing this paper I decided to canvass a group of progressive secondary schools which are now experimenting with the curriculum, in order to find out what they are doing about mathematics. The schools chosen were the thirty

[12] E. F. Lindquist, "The Gap between Promise and Fulfilment in Ninth-Grade Algebra," *School Review* (December, 1934), Vol. XLII, pp. 762–71.

[13] *Ibid.*, p. 769.

private and public schools of highest character and established reputation which a year or two ago were assured by two hundred colleges and universities of freedom from the usual college entrance requirements in order that they might experiment under the auspices of the Progressive Education Association. The experimental programs began in 1933–34. A year ago Dean McConn, in describing the inception of the experimentation said that "The Commission and Directing Committee did not prescribe any one plan or even propose alternative plans, but invited proposals to be originated and formulated by each school for itself. . . . In most instances, the content subjects will be moved into the foreground of the picture, with substantial enrichment of factual material. In a number of cases a core curriculum will be established centering in some one broad field of knowledge, usually the social studies, with which the work in other fields, including the tool subjects, will be closely integrated. . . . In practically all the experiments there will be increased correlation and interweaving of subject material. . . ."[14]

Of these schools I inquired, first, "Are you offering the usual mathematics courses? If so, just what is the offering in each grade and what pupils are required to pursue it?" Second, "Are you offering to any of your pupils a program of study in which mathematics is integrated with other subjects, such as science and social studies?" Seventeen schools responded, some giving quite detailed explanations of their plans for teaching mathematics. Thirteen schools indicated no integration of regular secondary-school mathematics with other subjects, while four signified varying degrees and types of integration—one starting a curriculum integrated primarily around social topics and acquainting pupils in the first year with the Greek and Latin origins of mathematics, one conducting a special course in tenth-grade geometry which seeks some integration of geometry with history, art, and architecture, one attempting correlation of mathematics and science, and one administering a curriculum organized around the main educational objectives without reference to subjects, with functional and practical mathematics coming in when problems demand it. In all or nearly all of the schools responding to the inquiry, it was plain that the experimental procedures were being applied to only a part of their students. Most of the schools offered the usual courses in

[14] Max McConn, "Freeing the Secondary School for Experimentation," *Progressive Education* (November, 1933), Vol. X, pp. 367–72.

secondary-school mathematics, seven emphasizing some integration of the several branches of mathematics. Six schools required some mathematics of all ninth-grade pupils, six said mathematics was elective in ninth grade, while five did not indicate whether the subject was required or elective. Only three schools required mathematics of all pupils in the tenth grade.

Viewing as a whole the reports of these schools, it is evident that mathematics—at least, mathematics beyond arithmetic—is pretty largely ignored in the experimentation with integrated curriculums. One mathematics head in a well known progressive private school stated that he was "able to find little correlation between mathematics and the rest of the curriculum." The situation is highly significant from the standpoint of modern educational theory, because the whole attempt to set up integrated curriculums is an expression of determination to create an environment in which the pupil has experiences which are more real and significant to him than those to be had from the school subjects.

I was somewhat impressed with an innovation in the Beaver Country Day School. The usual mathematics courses are offered in grades nine to twelve, but all are elective. The only required course is called "practical mathematics" and is offered twice a week in grades ten and eleven. The course was first introduced as an elective but proved highly popular because it met a definite felt need for more mathematics, and was therefore made a requirement. Attention is centered largely on such social topics as money and banking, thrift and investments, insurance, taxation. Undoubtedly the mathematics is mostly arithmetic, but it is probably arithmetic for which the pupils were not ready in junior high school years because they were not then ready to think in the situations demanding such arithmetic. This is a case of teaching a tool in situations demanding its use. High-school pupils should grow continuously in quantitative-mindedness, in seeing and in reacting to their expanding world with mathematical exactness.

Here, it seems to me, is an important cue for the fundamental reorientation of mathematics. Instead of studying quantity in the abstract, let there be increased attention to the quantitative aspects of what we call the content subjects—that is, those fields of knowledge which deal directly with the natural and the social environment. Let the pupil's major interest be the element of environment which is being viewed; let the minor interest be the instruments which one utilizes to obtain the completest view.

Report of the Commission on Examinations in Mathematics

I. INTRODUCTION

The Commission has completed its study of the problems submitted to it and embodies its conclusions in the recommendations which follow. Some general observations precede to indicate the fundamental points of view which have guided the discussions of the Commission, and which find expression in the recommendations (sections II and III). Some general comments for the guidance of teachers conclude the report (section IV).

1. The purpose of the examinations of the College Entrance Examination Board is to determine the candidates' fitness for college. As far as examinations in mathematics are concerned, the meaning of fitness for college varies among different groups of candidates. The Commission has found it essential to its purpose to recognize the following three groups of candidates:

(α) Those who do not intend to carry on in college the study of mathematics or natural science, but who base their claim to be admitted to college in part upon the study of mathematics in the secondary schools.

(β) Those who intend to fulfill at least the minimum college requirement in mathematics or natural science.

(γ) Those who look forward to more advanced undergraduate work in mathematics and science.

2. The mathematics examinations should be of such character as to furnish an adequate basis for a report to the colleges as to the fitness of the candidates in each of the groups mentioned in 1.

3. The examinations should be such as to determine:

(a) the candidate's understanding and appreciation of the fundamental principles and characteristic modes of approach of mathematics;

(b) his technical equipment and his knowledge of mathematical facts.

Emphasis on *both* of these aspects of the examinations must increase as we pass from group (α) to group (γ). The first must play an important part in all the examinations. The second will receive

greater emphasis in the examinations for groups (β) and (γ) than in the examination for group (α). Every test of the technical aspect of the subject should be designed to reveal not only the candidate's skill in formal manipulation of terms and symbols, but also the extent to which he has acquired a thorough understanding of the fundamental principles.

4. The examinations must therefore bring together all the mathematical subjects which the candidate has studied. Rather than separate tests in algebra, plane geometry, trigonometry, and so on, they should be comprehensive in character. It may be desirable to change from year to year the number of questions which each subject is to contribute to the examination.

5. The character of the examination must be such as to combine the advantages of the longer, essay-type, multiple-step question with those of the single-step question of the type now used in the mathematical section of the Scholastic Aptitude Test. The distribution of questions of these two types must in general vary from year to year.

6. The Examiners should be free to adopt any method that seems to them suited to the purposes of the examinations.

II. RECOMMENDATIONS

1. It is recommended that from 1936 on the examinations now known as A2, CD, Cp. 3, Cp. H, and the mathematical section of the Scholastic Aptitude Test be dropped; that examinations A, A1, B, C, D, and E be continued for a period not to exceed three years, so as to facilitate the transition to the new procedure, indicated in 2.

2. It is recommended that beginning in 1936 there be instituted three new examinations, hereinafter referred to as Mathematics Alpha, Beta, and Gamma. After 1938 these will be the only examinations in mathematics set by the College Entrance Examination Board.

3. It is recommended that the purpose of Examinations Alpha, Beta, and Gamma be that described in the following paragraphs.

(a) Examination Alpha is to serve as a basis for certifying to the colleges that a candidate has given evidence that he has profited by the study of mathematics in school to the extent of having acquired an understanding of mathematical procedure and an idea

of the structure of the mathematical sciences. But the Board is not prepared to say on the basis of Examination Alpha alone that the student is equipped to study mathematics or physical science in college.

(b) Examination Beta is to serve as a basis for certifying to the colleges that the student is prepared to carry on at least the minimum college requirements in mathematics or physical science.

(c) Examination Gamma is to serve as a basis for certifying to the colleges that the student has reached a degree of proficiency to justify his taking more advanced work in mathematics and the sciences.

4. It is recommended that the scope of the examinations be as indicated below:

(a) *Alpha.* The scope of the examination shall be arithmetic, elementary algebra, numerical trigonometry, and a brief survey of plane geometry. Further topics in algebra and geometry may be included in this examination, but it is to be so constructed that there will be no disadvantage to those who have studied only the elementary portions of these subjects. This examination is to test, in so far as this material allows, an understanding of the relation of mathematics to the environment, in particular of the significance of its method of approach to concrete problems and of the way in which the abstract concepts of mathematics are obtained from experience.

(b) *Beta.* The examination is to include plane geometry, elementary algebra, that part of algebra usually referred to as Quadratics and Beyond, and numerical trigonometry. Questions from solid geometry, from more advanced parts of algebra and trigonometry, and from elementary statistics can appear on this examination in the same manner as that described for the more advanced topics in Examination Alpha.

(c) *Gamma.*The core of the examination shall be trigonometry, solid geometry, and advanced algebra. There may also be questions in analytic geometry and the calculus; these questions are to occupy a place similar to that occupied by the more advanced parts of Alpha and Beta. It is understood, furthermore, that questions from more elementary parts of mathematics are not excluded.

5. It is recommended that the form of the examinations be governed by the following rules:

(a) Examinations Alpha and Beta shall be three-hour examina-

tions; Gamma shall be a four-hour examination (See the remark on the last page of this report.).

(b) All the examinations shall include questions of the type now used in the mathematical section of the Scholastic Aptitude Test.

(c) Book propositions and originals shall be put on the same basis.

(d) The examinations shall contain enough questions to measure the ability of the good student as well as of the poor student; it is, therefore, to be expected that no candidate will answer all questions.

(e) The questions shall be arranged in general in order of difficulty.

6. It is recommended:

(a) That questions to be used in Examination Alpha be tried out in experimental sections, as is customary at present in the mathematical section of the Scholastic Aptitude Test.

(b) That the papers used in Examination Alpha be kept secret, but that the Secretary's office supply upon request sample examinations which will, as far as possible, indicate the form and character of this examination.

(c) That as soon as feasible the papers for Examinations Beta and Gamma be prepared in the same way as Examination Alpha. This would involve trying out the questions experimentally and keeping the examinations secret, though the Secretary's office would supply sample examinations which would, as far as possible, indicate the form and character of these examinations.

(d) That the Examiners receive full information as to the results obtained from pretested material.

7. It is recommended that Documents 107 (Definition of the Requirements in Elementary Algebra, Advanced Algebra, and Trigonometry) and 108 (Definition of the Requirements in Plane Geometry, Solid Geometry, Plane and Solid Geometry) be replaced by a single document, to consist of the present report.

III. THE EXAMINATIONS

Examination Alpha

Examination Alpha will occupy not more than three hours. The scope of the examination shall be arithmetic, elementary algebra, numerical trigonometry, and a brief survey of plane geometry. Further topics in algebra and geometry may be included in this

examination but it is to be so constructed that there shall be no disadvantage to those who have studied only the elementary portions of these subjects. This examination is to test, in so far as this material allows, an understanding of the relation of mathematics to the environment, in particular of the significance of its method of approach to concrete problems and of the way in which the abstract concepts of mathematics are obtained from experience.

Some indication of the meaning and scope of these phrases is furnished by the following list of specific accomplishments which the candidate will be expected to have acquired as a result of his study of the fields of mathematics which were mentioned above.

1. Understanding of the importance of convenient symbolism for the communication of ideas.
2. Skill in arithmetical operations and elementary algebraic manipulation.
3. Knowledge and understanding of the facts of mensuration.
4. Understanding of functional dependence, its mathematical formulation and its applications.
5. Ability to apply a general rule to particular cases; e.g., the formula $(a+b)(a-b)=a^2-b^2$ to the determination of such products as 88×92.
6. Ability to put a proposition in logical form, i.e., indicate hypothesis and conclusion.
7. Understanding of what constitutes a mathematical proof, and of the nature of a mathematical definition, i.e., their connection with and dependence upon postulates and primitive concepts.
8. Appreciation of the nature of a deductive science.

This examination will supersede the mathematical section of the Scholastic Aptitude Test and will preserve the essential features of that test. The form and general character of Examination Alpha are illustrated by a specimen examination which may be obtained from the Secretary of the College Entrance Examination Board.

Examination Beta

Examination Beta will occupy not more than three hours. The scope of the examination represents the preparation in mathematics expected of students intending to do at least the minimum work in mathematics and the natural sciences required in colleges. It includes algebra, plane geometry, and numerical trigonometry. Questions from solid geometry, from more advanced parts of

algebra and trigonometry, and from elementray statistics can appear on this examination in the same manner as that described for the more advanced topics in Examination Alpha.

Besides giving evidence of having attained the development in his thinking and the limited technical skills which are tested in Examination Alpha, the candidate is expected to demonstrate in Examination Beta that he has acquired a sound knowledge of the content and procedures of algebra, plane geometry, and numerical trigonometry. Such knowledge cannot be based therefore on merely formal technique and memorizing of proofs, but must rest on a foundation of understanding of the principles.

Algebraic operations should be carried out not as memorized tricks but as instances of the fundamental processes which dominate elementary algebra. Much of the work in formal algebra can be unified by the adoption of a systematic point of view. A large part of elementary algebra consists in carrying out the rational operations (addition, subtraction, multiplication, and division) on expressions which have been obtained by performing these same operations on one or more variables. Recognition on the part of the candidate that such composite forms obtained by successive applications of the rational operations are *always* reducible to the quotient of two polynomials without common factors should result in greater mastery of the manipulative part of elementary algebra. In particular it should help in giving a more definite meaning to the term "simplify" as used, for example, in the removal of parentheses, in the reduction of complex fractions, etc.

In the geometric portion of the examination, originals and "book propositions" will be placed on the same basis. In his demonstration the candidate will be expected to employ a concise and accurate style of exposition which emphasizes important details but dares to ignore purely formal particulars.

Just as the power of the candidate in geometry is tested by originals not included in any standard list of propositions, so in algebra and in trigonometry questions on any topic, which go somewhat beyond the field described, but which are within the power of a candidate properly trained in that field, may form a part of the examination.

Sharp separation between algebra, geometry, and numerical trigonometry will not be maintained. Understanding of the interrelations between these subjects should be part of the candidate's

equipment. In a similar, but more restricted way, he should understand the relation between different theorems in geometry and between different groups of theorems.

A detailed specification of the topics to be covered by the examination is not desirable. They may vary from year to year; it is indeed intended to leave the Examiners free to use such material as may appear to them best suited to the purposes of the examination. The following list serves therefore to indicate the general scope of the examination and the point of view from which the material is to be considered rather than to provide a catalogue of topics.

Algebra

1. Language, ideas, and processes of algebra.

(a) Symbolism and vocabulary (e.g., coefficients, exponents, use of parentheses, use of subscripts).

(b) Number systems (e.g., natural numbers, signed numbers, rational, real, complex numbers).

(c) Laws of operation (e.g., rational operations in various number systems).

(d) Techniques (e.g., the summing of progressions, the use of the binomial theorem).

2. Functional dependence.

(a) The formula (e.g., formulas which appear in geometry, in physics, or in other fields within the experience of the candidate).

(b) The equation (e.g., linear and quadratic equations in one variable, systems of equations in several variables, their connection with the identity and with the formula).

(c) The graph (e.g., representation of statistical data and of relations between two variables given by equations).

3. Problem solving, i.e., the translation of a question into algebraic form, its solution and interpretation.

Geometry

1. Congruence of rectilinear and of circular figures.
2. Perpendicularity and parallelism of lines.
3. Comparison and measurement of angles in rectilinear figures and in circles.
4. Measurement of areas in rectilinear figures and in circles.
5. Similarity of rectilinear figures and of circular figures.

6. Fundamental constructions by means of straight-edge and compasses.

7. Locus theorems and problems.

Trigonometry

1. The solution of right triangles by the use of four-place tables (natural functions and logarithms).

2. Some understanding of the technique of calculation, of the importance of checks, and of the limitations imposed upon the accuracy of results by the use of approximate data and of tables.

Examination Gamma

Examination Gamma will ocupy not more than four hours. The scope of the examination represents the equipment expected of candidates for technical schools and the degree of development attained by students who have an interest in and aptitude for mathematics and the natural sciences, such as would lead them to pursue more advanced study of these subjects in college. The examination will test the candidate's knowledge of trigonometry, solid geometry, advanced algebra, and—to a lesser extent—analytic geometry and the calculus. It may also include questions on elementary algebra and plane geometry. Although most candidates will have devoted only a year to the study of advanced mathematics, the examination will cover a somewhat wider range of topics than is ordinarily compassed in that time.

The examination will be so constructed that one year of study—or the time necessary for adequate preparation to meet the former requirement Cp. H—will prepare a candidate similarly to answer approximately five-sixths of the questions. These will be arranged for the most part in order of increasing difficulty and will be divided among the different topics as follows. Approximately one-third of the questions will deal with trigonometry, one-fourth with solid geometry, one-fourth with advanced algebra, and one-sixth with analytic geometry and the calculus together. It will be noted that the division of questions on the examination between the subjects trigonometry, solid geometry, advanced algebra, and analysis will be roughly proportional to the numbers 4, 3, 3, 2 and that on the same scale a year's study will be represented by the number 10. The questions on trigonometry, solid geometry, and advanced algebra may be concerned with any one of these three subjects, with any two of them, or with all three.

The examination will in general not observe strict separation of the different subjects. Candidates will be expected to be able to use any one of the subjects they have studied in the solution of a problem with which they are confronted. Technical facility founded on and related to the understanding of basic ideas is the accomplishment which the examination will test.

Proofs which involve the theory of limits are not considered to be within the scope of this examination. For a good many propositions in solid geometry, in the theory of equations, and in the calculus, such proofs are necessary. It is intended that such propositions be taken as postulates, and that their proof in terms of a simpler set of postulates be looked upon as one of the objectives of more advanced work.

The examination is designed to test candidates of different preparations without defining any particular course of study. It seems desirable, however, to interpret the scope of the new examination by comparing it with that of the former Cp. H examination. It was possible for candidates to prepare for that examination by dividing a year of study between two of the three advanced subjects, trigonometry, solid geometry, and advanced algebra. Almost invariably one of these two was trigonometry, and it is assumed that the schools will continue their instruction in trigonometry as heretofore; but instead of devoting half a year to solid geometry or advanced algebra alone, they are encouraged now to teach both solid geometry and advanced algebra. In most schools for the present this will mean about three-tenths of a year in each of these two subjects.

Although the Examiners will not be bound to adhere literally to any particular syllabus, they will base their questions normally on the following topics.

Trigonometry

1. The six trigonometric functions of angles of any magnitude. The computation of five of these functions from any given one. Functions of 0°, 30°, 45°, 60°, 90°, and of angles differing from these by multiples of 90°.

2. Determination, by means of a diagram, of such functions as sin (A+90°) in terms of the trigonometric functions of A. Use of trigonometric tables, with interpolation.

3. Circular measure of angles; periodicity; length of an arc in terms of the central angle in radians.

4. Solution of right and oblique triangles (both with and without logarithms) with special reference to the applications. Value will be attached to the systematic arrangement of the work.

5. Derivation of the Law of Sines and the Law of Cosines.

6. Proofs of fundamental relations and formulas, such as the formulas for tan $(x+y)$ and cos $2x$, and of simple identities derivable from them.

7. Solution of simple trigonometric equations.

8. Theory and use of logarithms. Solution of exponential equations.

Solid Geometry

It is expected that the candidate will have facility in visualizing figures in space and in representing such figures on paper; that he will know the terminology and the principal facts and formulas of plane and solid geometry; and that he will have studied the derivation of many of these formulas and the proofs of the important propositions. In general the examination will make a greater demand on his appreciation of three dimensional relations than on his ability in formal demonstration.

Questions on the examination which call for formal demonstration will be limited to theorems concerning the relations of lines and planes in space. For this purpose "book propositions" and originals will be considered to be on equal bases. The candidate will be expected to employ a concise and accurate style of exposition which emphasizes important details but dares to ignore purely formal particulars.

Inasmuch as the scope of this part of Examination Gamma is necessarily much more restricted than that of the former examination in solid geometry, some indication is given in the following paragraphs of what this scope is to be; it is not the intention to give a precise definition.

1. The relations of lines and planes in space. Of particular significance are the theorems which deal with parallelism and perpendicularity of a pair of lines, a pair of planes, a line and a plane; the concepts of skew lines and of the angles determined by them.

2. The properties and measurement of prisms, pyramids, cylinders, and cones.

Although the candidate will not be asked to derive those formulas concerning cylinders and cones which require the method of limits, it is assumed that he will have been led to appreciate the

validity of these formulas regarded as limiting cases of corresponding formulas concerning prisms and pyramids.

There will be little emphasis upon the logical aspect of the mensuration of the solids mentioned above; it is expected, however, that the candidate will have noted the logical sequence in the theorems leading from the formula for the volume of a rectangular box to the formula for the volume of a triangular pyramid.

3. The sphere and the spherical triangle.

The Examiners will wish to count on the candidate's appreciation of the distinctive rôle played by symmetry in spherical geometry, but will not ask for proofs of theorems concerning symmetric spherical triangles.

The candidate will not be asked to derive the formulas for the area and volume of a sphere; it is expected, however, that he will have noted the logical connection between polyhedral angles and spherical polygons and will appreciate how the formulas for the area and volume of a sphere can be regarded as limiting cases of other mensuration formulas.

4. The solution of locus problems, to the extent of visualizing, describing, and representing the figure on paper, is explicitly included; formal proofs will ordinarily not be required.

Advanced Algebra

1. Elementary aspects of the theory of equations with emphasis on the following ideas: The theorem that an equation of the nth degree has n roots, if every such equation has one root. The remainder theorem and its applications. The coefficients as symmetric functions of the roots. Condition for a rational root. Conjugate complex roots of equations with real coefficients. Descartes' Rule of Signs. Simple transformations of equations.

Approximate solution of numerical equations. Preliminary location of the roots by the graph. Determination of the roots to two or three significant figures, preferably by some less elaborate method than Horner's Method.

2. A numerical and geometric treatment of complex numbers, including the sum, difference, product, and quotient of two complex numbers in the form $a+bi$.

3. Permutations and combinations, restricted to the case of n objects all of which are different. Probability restricted to problems of moderate difficulty.

The examination will contain no questions on determinants, simultaneous quadratics, scales of notation, or mathematical induction.

Analytic Geometry and the Calculus

1. The analytic geometry of straight line and circle. The distance between two points, the slope of a line, parallel and perpendicular lines; the equation of the circle and its relation to center and radius. Other simple loci.

2. The differentiation of algebraic functions with applications to slopes, maxima and minima, rates of change, velocity, acceleration. With respect to fundamental notions concerning the derivative, the candidate should distinguish clearly between definition, assumption, and proof and should have an accurate knowledge of the rôle of the limiting process.

IV. GENERAL COMMENT ON THE EXAMINATIONS

In formulating its recommendations, the Commission has been strongly influenced by the wish to leave teachers of mathematics in secondary schools free to guide the development of their pupils in such ways as seem to them most desirable.

It has been the aim of the Commission not to indicate the content of the mathematics courses which lead up to the examinations in such detail as to hamper the work of the teacher, but to be sufficiently definite in specifying the scope of the examinations to avoid uncertainty on the part of the teacher.

It follows from these considerations that there is no reason why teachers in their choice of material should not go beyond the fields indicated in the definitions.

The examinations will attempt moreover to determine to what extent the candidate is able to use the logical thought processes, acquired in his study of mathematics, in other fields.

Every subject in mathematics, if well taught, contributes, though perhaps not always to equal extent, to the understanding of the fundamental principles and the characteristic modes of approach of mathematics. Technical skill in any part of mathematics is enhanced by the study of more advanced portions of the subject in which the earlier parts enter as tools.

The examinations will include questions which not all the candidates are prepared to answer. It is not expected that every can-

didate will answer all the questions. In particular, Examination Gamma is intended to be finished in three hours by the majority of the candidates; but four hours will be allowed so that candidates of unusually wide preparation will have the opportunity to show the full extent of their ability. Each examination will be in effect a contest in which no one is expected to obtain a perfect score, but which will serve nevertheless to rank the contestants in order of merit.

It is the intention so to frame the examinations as to indicate clearly whether or not the candidate belongs in one, or several, or none of the groups (α), (β), (γ) into which the report has divided those who apply for entrance into a college.

Respectfully submitted,

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Be sure to order the Tenth Yearbook on "The Teaching of Arithmetic" now. See page 134 of this issue for a complete description of the book.

How Shall We View Elementary Education as Regards

1. Discipline
2. The Psychology of Learning[1]
3. Subject-matter?

By WILLIAM C. BAGLEY

In the consideration of the three topics listed for discussion I shall give practically all of the time at my disposal to the second topic, Subject-matter, because at the present time the chief problems of educational theory and practice seem to be centered here. My own remarks concerning the present status of discipline and of psychology will consequently be very brief. I shall deal first with these and for the sake of brevity I shall take the very great liberty of being somewhat dogmatic.

Discipline

Discipline I shall regard both as a series of learning experiences and as the outcome of this series—both as a process and a product. My assumption is that effective living in a social environment necessarily involves certain acquired traits which are in the nature of predispositions. Conspicuous among these are willingness to cooperate in common enterprises and willingness to live conformably with the social standards which are obviously essential to the welfare and survival of the group. These are so nearly universal in all peace-groups and they are so clearly essential to social life that they may be regarded as enduring values in a world of change. Essentially these standards represent on the one hand the rights which society grants to the individual and on the other hand the duty of the individual to respect these rights in others. Quite obviously the school as a social community affords numerous experiences which in many instances develop these traits. The learner becomes accustomed to social living and to its necessary condi-

[1] Prepared for a joint discussion with Professor William H. Kilpatrick, Teachers College, Columbia University, January 11, 1935. The occasion was a conference on the problems of elementary education arranged by the elementary-education group of Teachers College with especial reference to the needs of teachers and administrators in the metropolitan area.

tions. Equally important, perhaps, is the regimen of order and industry that the well managed school reflects, for both order and a capacity for sustained effort are acquired traits which the race learned very slowly but which have been fundamentally essential to social progress. My point is that the life of the school may here be a most effective educational agency.

The disciplinary influences of the school depend very largely upon collective opinion, and the collective opinion of the pupils' fellows is a most potent force. To lose caste among one's fellows is in many ways the severest of all penalties. Unguided and unenlightened, the discipline of a pupil by his fellows not only may be far more inhumane than the older types of punishment but may also approve what should be condemned and condemn what should be approved. The problem for the teacher would seem to be the development of a school morale which will insure the direction of this powerful force of social approval and condemnation toward the ends for which the school exists. I formulated the problem twenty years ago as the development in the pupil-group of a "fashion" that will so to speak "take for granted" courtesy, helpfulness, industry, and order.

The Psychology of Learning

The present situation regarding psychology as a basis for educational theory and practice is complicated (1) by the apparent breakdown of the psychologies which have explained learning on the basis of assumed specific connections among the neurons; (2) by the multiplication of psychological systems or schools; (3) by radical inconsistencies in the findings of controlled experiments which have attempted to answer very simple questions regarding the relative value of different teaching procedures; (4) by increasing sketpcism regarding the validity of conclusions drawn from the results of mental tests; and (5) by the dominant and rapidly expanding influence of the Progressive educational theories which seem now definitely opposed to any efforts to test learning by anything akin to objective methods.

Personally I believe that an educational theory to be worthy of attention must have back of it at least intelligible assumptions regarding the nature of the learning. I am also inclined to the belief that learnings on the more primitive levels are subject to laws that some time will be satisfactorily formulated. Furthermore I believe

that the measurement of the products of such learnings by the present instruments is far better than no measurement at all.

It is the higher emergent learnings, however, that are most important in education, and these present-day psychology has in some cases entirely ignored and in other cases attempted to explain in the same way that it explains primitive or lower-level learnings. The very great significance of the higher emergent learnings is spectacularly reflected in the recent advances that have been made in astronomy and physics, not initially through observation and experiment, but through the medium of pure mathematics. As a recent writer has put it, reasoning has so far outstripped perception and concrete imagination that truths predicted by pure reason and then confirmed by observation or experiment reveal conditions that completely baffle the effort of mind to conceive of them in concrete terms. Scientific psychology has assumed that reasoning is a capacity which grows out of concrete, empirical experience through processes that may at least be imagined. But the fourth dimension and $\sqrt{-1}$ and certain exemplifications of the quantum theory defy the efforts of the most agile imagination to picture them visually or through any other sensory medium. This is reminiscent of the conclusion reached by Immanuel Kant a century and a half ago; namely, that the *forms* of thought are not derived from sensory experience, but are among the "given" factors in mental life; hence the famous Kantian term, "pure" reason. Of late, physics has been impelled to return to metaphysics in an attempt to clarify some of the assumptions made necessary by its recent discoveries; and it is fairly predictable that psychology will of necessity follow the same course.

These last statements may have little to do with any possible contributions of psychology to elementary education. And yet it is in the elementary school that the start is or may be made toward the higher emergent learnings. Aside from the utilitarian values of simple measurement and computation, the study of arithmetic may well involve an initial glimpse of the meaning of number in its broader and more truly mathematical significance as what Judd calls "a device for arranging experiences," the conception of which he regards as "one of the most general of all generalizations."[2]

[2] Judd, C. H.: "Informational *vs.* Computational Mathematics," *The Mathematics Teacher*, April, 1929, pp. 187 ff.

Subject-matter

We turn now to subject-matter. I am surprised that even the word is permitted in this program. There is no term—not even discipline—that is in greater disrepute in educational theory and practice to-day. The severest epithet that one can hurl at a teacher or an administrator is to say that he is "subject-matter conscious." I have not discovered what the antithesis, or corresponding virtue, really is. If "subject-matter consciousness" is the vice, is unconsciousness of subject-matter the virtue—or perhaps innocence of subject-matter—since innocence and virtue are closely associated? I am left myself in the predicament of the student of the new physics; I cannot comprehend education without subject-matter;—but perhaps I could if I had command of the proper symbolism.

In the present paper the subject-matter of education will be regarded primarily as the experience of the race. It is true that the direct personal experience of the individual is a powerful moulding force. Spontaneous play has long been recognized as an important factor in physical and mental development. In the course of undirected experience, too, the individual may stumble upon some important truths that it took the race a long time to learn. This is especially noteworthy among the learnings that are clearly illustrated in the material environment as modified by Man, and in the customs and standards, the mores and folkways, of the people In these cases, however, while the learning-experience of the individual may be undirected, it is actually the experience of the race that constitutes the subject-matter of the learnings. In any event, we all agree, I think, that in the early education of the school as well as of the home there should be abundant opportunities for spontaneous play and for activities that are not consciously directed toward a predetermined learning goal, and we all agree that ideally there should be for every child an environment reasonably rich in stimuli that will encourage explorations and observations and excursions and experiments of many and varied types.

Such opportunities, however, important as they are, cannot be a substitute for that type of education which aims specifically to transmit to the learner those elements in the cultural heritage of mankind which give meaning and significance to the world in which we live and which have made possible some measure of control over natural forces. This conclusion is based upon the assumption that the outstanding and unique human prerogative is the capacity to

accumulate, refine, and transmit learnings. The progress of Man as Man over tens of thousands—and probably hundreds of thousands—of years has been determined fundamentally by this capacity. The primitive types of education, based upon the activities of family life, the religious ceremonies and collective enterprises of tribal life, and later (with the division of labor) the apprenticeship system, sufficed over thousands of years to keep alive the standards or mores essential to social living and such primordial arts as the conservation and later the kindling of fire, the techniques of hunting and fighting, preparing food, and fashioning crude weapons, tools, and implements from clubs and stones and bones, and later from copper and bronze. It was with a slowness almost incredible to modern Man that learnings were increased in kind and refined in quality. Even after Man had acquired essentially the biological equipment that he possesses to-day, not only centuries but millennia elapsed before his subsistence depended upon anything less precarious than the chase, before his shelter advanced beyond the caves and the overhanging rocks which Nature had provided, before his clothing was anything more than skins stripped from the carcasses of wild animals.

This is the background against which the earliest civilizations must be projected, and against this background these civilizations stand out with startling brilliance. The initial condition of civilization in the current usage of the term is the possibility of insuring a relatively permanent and non-precarious food-supply capable of supporting large, settled populations. This condition seems first to have been met in the marvelously fertile, readily irrigable, and easily protected valleys of the Nile and the Tigris and Euphrates. Given this condition, the perpetuation and further evolution of these civilizations depended upon two fundamental social arts: (1) measurement and computation, and (2) phonetic writing. Professor Thorndike once said that, if a tribesman of the Stone Age had been able to count accurately to one hundred, he might soon have come to possess most of the property of his tribe, so great would have been his advantage over his fellows. Literacy, both numerical and linguistic, was a major starting-point of progress as we understand progress to-day. Literacy in this double sense of the term meant a vast improvement in the accumulation, refinement, and transmission of learnings. It meant the dawn of pure and applied science through the parallel development of math-

ematics, astronomy, surveying, engineering, and architecture. It made possible the evolution of law and justice. It was a prime condition of the growth and influence of great religions, some of the very earliest of which, as Breasted has recently shown, stimulated the human mind to a recognition of social responsibility and led to that emergent quality which we call conscience, the appearance of which, he assures us, was the result of the social experience of man himself and "was not projected into the world from the outside."[3]

A written language and an efficient number system have been indispensable possessions of subsequent civilizations. This does not mean that these civilizations were based upon universal literacy; it does mean, however, that a sufficient number of persons in each generation were so equipped that they could keep alive the lessons that the past had learned. It was actually to the advantage of the ancient rulers that literacy should be limited to a group small enough to be controlled. Perhaps an important factor in the power of the Egyptian kings was the fact that they alone could publish, so to speak, the predictions as to flood-time and seed-time and harvest which the literate scholars attached to their courts were able to make in a degree of accuracy which convinced the people that their Pharoah was no less than God himself. Efforts to restrict literacy and to prevent the enlightenment of the masses have been features of nearly all of the civilizations since that time. In fact, the development of relatively universal literacy did not come in any large country prior to the nineteenth century. To-day we recognize the wide influence that lies with those who monopolize natural resources and use them for their own selfish ends. But such an influence sinks into insignificance in comparison with that which would accrue to a monopoly of learning.

It is reasonable to believe that the restricted literacy of the earlier civilizations played an important part in their ultimate decline and fall. All civilizations have found the school, or something equivalent to the school, essential to the conservation of literacy and of the kinds of learning that it produces. To the preservation of other learnings, such as the arts of agriculture, the skilled trades, and the handicrafts, the informal and largely incidental education of family life, group life, and occupational life sufficed—but literacy

[3] See Breasted, J. H.: *The Dawn of Conscience*, p. xv; also ch. i.

and its products required an organized agency which could provide an extended, systematic training. When this training failed, civilization failed; or when civilization succumbed to conquest from the outside, the training failed. Cause and effect are of slight significance here; the result was the same in either case.

A spectacular procession of civilizations, separated sometimes by long periods of reversion to barbarism, constitutes the ancient history of Mesopotamia. Beginning with the Sumerian, which was almost contemporaneous with the dawn of civilization in Egypt, the brilliant list includes the Babylonian, Assyrian, and Chaldean, and ends with the Abbyssid Arabic supremacy which flourished at a time when Western Europe was under the cloud of ignorance and superstition and mental stagnation that so quickly followed the fall of Rome. Baghdad then was the world's largest city and the world's center of learning. Letters and science and art were cultivated, and not only was a great university developed but it is said that there was even an approach to universal literacy. Conquest after conquest from the East, however, reduced the rich plains of Mesopotamia once more to a desert, and the fruits of Arabic learning only gradually found their way into Western Europe. Among them were contributions of outstanding significance: the Arabic refinements of the Hindu number system; the development of spherical trigonometry and the familiar trigonometric functions; most substantial advances in medicine and surgery; as well as priceless translations of, and commentaries on, the works of Aristotle and other Greek scholars. It was the influence of the Arabic culture, largely exerted through the contacts of the Crusades, that finally aroused Europe from the intellectual torpor under which it had not lived but merely vegetated for more than half a millennium.

Nor is there a more impressive illustration of the rapidity with which the fruits of a civilization may be lost through the loss of literacy than this same period of European darkness. Less than two hundred years after the fall of Rome even the tradition of its glory had been forgotten. Men and women and children lived in the shadow of the Coliseum or gazed upon the tomb of Hadrian, dimly wondering what these structures meant. The debris of ignorant and filthy generations covered and buried deep the Forum. Even the church, which had frowned upon secular learning, almost forgot the medium in which its own annals and doctrines were

recorded. It is said that one of the popes, suspicious that serious losses had been sustained, traveled in disguise to one of the Arabic universities to find whether his fears were justified. Throughout western Christendom the priests were commonly illiterate and mumbled from memory the prayers and litanies that had for them as little meaning as for the communicants to whom they ministered. Charlemagne himself, innocent of letters in his early life, was none the less quite too intelligent not to recognize their value. Yet tradition has it that when he decided to establish a school at his palace not a competent teacher could be found in all France, and it was necessary for him to import from England the monk, Alcuin.

There would be no need to review these well-known facts were it not for the perennial tendency to minimize the importance of the long experience of the race as the primary basis for the organized education of each generation. From its inception in the earliest of the civilizations the school has been far from perfect. It has been subject to the limitations inherent in its materials. It could rise no higher than its source. Inevitably it transmitted error as well as truth, especially in those phases of human experience which, in the absence of truth, became overlaid with superstition or open to fraud and exploitation. Furthermore the very efficiency of literacy led to an emphasis of the letter and a neglect of the spirit of learning, to a far too great dependence upon memory as compared with reason, to an unfruitful preoccupation with linguistic disciplines such as grammar, rhetoric, and oratory. Against these evils and exaggerations there has been an age-old and ever-recurring protest. There has been an incessant demand for realities; for a "return to nature"; for a school that would shape its instruction around the interests of the learner; for a release from the bondage of the past. Each generation has had its educational leaders who rediscover these weaknesses and demand these and similar "reforms."

In the main, of course, the efforts of the critics and reformers have exerted a salutary influence, but in cleansing the educational domicile there has been a tendency to throw the baby out with the bath—to condemn out of hand the primary function of the school by elevating the claims of individual experience at the expense of the cultural heritage. A German historian, writing in 1859 concerning a group living in the 17th century who actually dubbed themselves the educational "Progressives," describes some of their ten-

ets as follows: "Estimating all communicated knowledge at a low rate, [these Progressives] preached to the younger generation the doctrine that they were to take pride in shaping out and accomplishing everything for themselves." "While undervaluing the receptivity so natural to youth, they endeavored to stimulate the learner to an incessant effort after precocious production." On the part of many there was "an undervaluation of history, and a deification of the present and the actual." "Since our method was comparable to nature," said the Progressives, "the children will learn voluntarily with ease and pleasure" and "all punishments would cease of themselves." Apparently there were schisms in the camp even in those days, for the author remarks that there were some traits that all Progressives had in common and particularly this, "They all vigorously controverted the systems of education prevailing in their day.[4]

I have suggested that, justified as many of their criticisms have been, the educational reformers have commonly tended to deprecate the educational values claimed for the cultural heritage of mankind. Certain of the great historic reformers—Erasmus, Montaigne, Milton, Comenius, and (to some extent) Pestalozzi—did not make this mistake, but the teachings of others have been so "progressive" as actually to be absurdly reactionary. You will recall that Rousseau would not have Émile very early in life use telescopes or microscopes, for, as he said, before Émile uses these instruments "I intend that he shall invent them, and this of course will probably take some time." It probably would. An error common to most of the historic educational reformers is that they have grossly underestimated the generations of time and energy and blind groping and repeated failures which have been the price that the race has paid for most of the learnings which they speak of so lightly and nonchalantly. Rousseau, if pressed hard enough, would probably have had Émile invent the phonetic alphabet before he used it,—a Mount Pisgah to which the race even after attaining its present type of brain did not climb until after some forty-five thousand years of blind wandering in an intellectual wilderness.

There is a simpler story not at all in the grand style—a story which may be apocryphal but which points the same moral and

[4] From a translation of an article by K. von Raumer, "The Progressives of the Seventeenth Century," in Barnard's *American Journal of Education*, March, 1859, pp. 463 ff.

adorns the same tale, and without, I think, doing undue violence to the probable truth. In the fifteenth century, it is said, a German merchant wishing to prepare his son properly for a commercial career went to a German professor and asked where his son could learn arithmetic. The professor replied that, if the father would be content to have his son master addition and subtraction, suitable instruction could be found in the German universities, but if he desired such advanced phases of the subject as multiplication and division he would have to go to Italy. This was not because the Germans were unduly stupid but rather because at that time the computation of multiplication and division was very complicated and difficult. We teach these skills now in the fourth grade to practically everyone. Just as the idea of position made the Hindu-Arabic number system far easier to manipulate than the Roman number system, so an apparently trifling invention such as the Austrian method of long division was in reality a long stride forward. It is literally astounding how apparently slight refinements of earlier learnings may readily facilitate the learning process and extend it to persons who would have been incompetent to earlier procedures. In another generation we may find Einstein's theory of relativity in the curriculum of the sixth grade;—that is, if we have a curriculum in the sixth grade or anywhere else in another generation!

For several years I suggested in my classes that in the preparation of teachers, especially for the elementary school, some attention might well be given to the history of what we once regarded as the fundamental subjects of school instruction. I recalled the rather heavy weather that I had made myself as a young teacher in attempting to teach reading and writing and the rudiments of arithmetic to little children, and it occurred to me that my labors, if not lightened, might at least have been enlightened, if I had been thoroughly conscious of the dramatic part that these arts have played in social evolution and of the slowness with which the race learned them. It is at least a decade since I was shown the error of my ways by one of my students. He came to me after I had delivered what I thought to be an especially convincing tribute to the phonetic alphabet. He said in effect: "I have been as patient as I could be with your vagaries, but when you suggest that elementary-school teachers should be taught to respect the subject-matter of the elementary-school curriculum I am through. I shall withdraw from the course." And he did.

This illustrates a tendency in contemporary educational theory which has a direct bearing upon our present discussion. I refer to the current teaching of educational theory to the effect that the curricula of the lower schools not only have been deficient in content and organization, but have actually been subversive of social progress. Favorite charges are that they have perpetuated the *status quo;* that they promote a "static" society; that they impose upon each generation the "dead hand" of the past; that they reflect the "authoritarian" control of vested selfish interests.

It is my belief that sweeping and undiscriminating charges which deliberately make use of terms that have acquired highly emotionalized meanings are a most serious handicap in our collective thinking regarding issues that are fraught with fundamental and far-reaching social consequences. This use of terms in educational discussion is just as reprehensible as is the use, for example, of the terms "bolsheviks" and "reds" by those who would stifle a rational consideration of the advantages and disadvantages of Communism as a social and economic order.

Take, for example, the term "authoritarian." The slightest reflection is sufficient to prove that an enlightened system of education must in large part reflect authority. It should not reflect an authoritarian control by vested interests, but it should reflect the authority of tested human experience. Against the authoritarian pronouncements of Hitler and his followers anent the alleged supremacy of the so-called Aryan race, I personally accept the authority of tested human experience which convinces me that this alleged race is a myth and that its proper place is in Mr. Bullfinch's well-known depository of fantastic untruths and not in the collective thinking of a potentially great people. Against Mr. Voliva's authoritarian dictum that the world is flat I accept the authority of tested human learnings, set forth first on *a-priori* grounds by the Greek and Arabic scholars and since then more than abundantly confirmed by empirical tests. And while it is true that apparently well-tested human experience has often been subject to error, I shall provisionally accept the authoritative hypotheses of competent physicists and astronomers regarding the constitution of atoms and the nature of the universe rather than return to an older authoritarian cosmology.

Then take this recently exploited bug-a-boo, the *status quo*, which is used in educational discussions quite without qualification or discrimination, and with the implication that our present social

status represents nothing that should be relatively stable and enduring. Let us see. Freedom of speech and freedom of the press are part and parcel of the *status quo* in democratic countries. They have been relegated to the limbo of outworn and discarded standards in Italy, Germany, Russia, and other countries that have abandoned democracy. I prefer to retain them, at least for a while. Similarly our *status quo* favors the use of ballots as against bullets in bringing about desirable changes in the social and economic structure. I am far from maintaining that violence is never justified, and my prejudice in favor of free speech in our present *status quo* would preclude me from denying the right of anyone to present the arguments in favor of a violent revolution at any time. But at present ballots seem to me to be preferable to bullets. For example, I am strongly convinced on rational grounds that the deviations from our former *status quo* which are in the direction indicated by outstanding policies of our present administration are also in the direction of progress. The overwhelming indorsement of these policies by the electorate last Fall intensifies my conviction that, at the present juncture, reason will be adequate and violence unnecessary.

In connection with curricular policies two other terms are coming to reflect an increasingly emotionalized content. On the one hand we have the term "integration"; on the other hand, the term "compartmentalization." At basis, the prejudice that has been built up against the types of teaching and learning that follow the internal relationships of the several subject-matter fields rests upon the assumption that the value of earlier human learnings to the present generation lies exclusively in their direct and overt application to present problems. As a consequence only the earlier learnings that can be thus applied are held to be worthy of a place in the program of education, and the one legitimate method of teaching them is in the context of immediate problems that appeal to the learner as meriting solution. As someone has suggested, these earlier learnings are merely tools, and when one needs a tool one goes to the shop and gets it. Under this assumption, not only the systematic and sequential mastery of race-experience is ruled out but also any programs of learning that are planned in advance. All learnings should awaite the emergence of problems or purposes to the solution or realization of which they may be applied.

One may readily grant the validity of this theory as a dominant

theory in the very immature stages of learning and in the learnings of those who remain permanently immature. One may grant as well the large value of such learnings as a supplement to systematic and sequential mastery even on the more mature levels. I maintain, however, that this theory as an exclusive theory is both unsound and unjust. It is unsound because it neglects a function of knowledge which, in comparison with the instrumental function, is at least of equal importance. This function is quite adequately expressed by a term that is happily acquiring a vogue in our discussions,—the term "background." It is the background of experience that determines the meanings which we read into the impressions that impinge upon consciousness, and which often determine whether a given situation constitutes a problem that should, if possible, be solved. This audience and my good colleague have a different meaning to me and constitute a different problem than they would to a savage from New Guinea—a cannibal, let us say, for the sake of emphasis. The falling of the apple had a different meaning to Newton and constituted a different problem than it would have meant and constituted to the same savage,—or to me for that matter. The existence of malaria on a high and dry plateau of the Andes became a problem to Darwin; to the natives it was scarcely a problem in the real sense of the term; it was a visitation of the evil spirits and to be accepted as such. To determine whether the laws of storms operate over the open ocean as they operate over the land was a problem to James H. Kimball, and its solution made it possible for him to give Lindbergh a trustworthy word "to go."

The important point, of course, is that these backgrounds of experience which determine meanings which in turn determine problems include not only direct, personal experience, but also experience that has been derived vicariously from the learnings of the race. Even Newton's genius in all probability would not have enabled him to detect a fundamental problem in the falling of an apple if he had not been equipped with the learnings and hypotheses derived from preceding studies; Darwin in his turn was puzzled because malaria in a non-humid climate was inconsistent with the learnings and hypotheses that then prevailed; and if Kimball had not been familiar with the laws of storms as they operated over the land the problem of oceanic storms would doubtless never have occurred to him.

The systematic and sequential mastery of past experience as

organized in the various fields of human inquiry I regard as the most dependable source of helpful backgrounds. It is easy to deride such mastery as the accumulation of mere information; it is easy to discredit learning processes that demand effort and sustained attention in the face of desires and distractions. One may be sure that any proposal which sanctions and rationalizes the lines of least resistance will receive the applause both of the gallery and of a goodly section of the pit.

But to sequential learnings, properly taught, the term "mere" information does not apply. Is it a "mere" fact that James Watt, after long struggle and failure, finally learned how to make the power of steam turn a wheel; or that Arkwright and Hargreaves, after overcoming similar difficulties, invented machinery which made possible the manufacture of textiles on a quantity basis; or that Eli Whitney finally succeeded in making the large-scale production of cotton possible? Considered each in and for itself these inventions might have been interesting but not fundamentally significant; but when they are put together they are seen to have initiated a movement which started social evolution on a new course and led to a new configuration of social forces. Not to have this knowledge as a part of one's present mental background is simply not to have the basis for interpreting some of the most significant present-day social and economic phenomena.

To read out of the picture systematic and sequential learnings based upon the logical, chronological, and casual relationships of the materials of learning is not only to obscure the importance of dynamic and dependable mental backgrounds; it is to encourage an ineffectiveness which has been clearly apparent whenever the theory in question has been applied consistently on a scale large enough to show how it works. The most spectacular instance, of course, was the twelve-year experience of the Soviet schools which led to a complete rejection of the theory by the edict of the Central Committee of the Communistic Party in August, 1932, and to the reorganization of the school programs on the basis of what the edict referred to as "systematic and sequential" instruction. Especially pertinent to our present discussion is the explicit reference made to the futility of teaching the social studies on the basis of present-day problems without an adequate *background* in history and in geography as such. And in the new programs these two subjects appear in their own right and with their own textbooks. (The use of

any textbooks had been proscribed in the earlier régime.) It is interesting to note that the educational authorities refer to these sweeping changes as an educational revolution, not a "reaction," and of the reorganization as representing a "new" movement, not a return to the old. The Soviet leaders, like some of our own leaders, know how to use words that will carry the desired meanings!

Other instances could be cited of the breakdown of the integration theory in practice. A widely heralded experimental college, based upon the integrationist doctrine, was abandoned, it is said, largely because the students discovered that they were missing certain learnings that their fellows in systematic courses were getting. This and other experiences, however, do not mean that the integrationists have nothing of value to offer. There is a clear place for efforts to relate the learnings in different subject-matter fields to one another, and to the problems of human life viewed on the largest possible scale; but both theoretical and practical considerations suggest very strongly that, with certain exceptions on the lower levels of learning, integrative learnings are most effective when there is something to integrate.

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Adrien-Marie Legendre

Born at Toulouse, September 18, 1752
Died in Paris, January 10, 1833

ALTHOUGH Legendre was born in the south of France, he spent the greater part of his life in Paris. He attended the Collège Mazarin in that city and, in 1775, he was made professor of mathematics at the Ecole Militaire where Laplace was also teaching. Legendre kept this position until 1780 when he resigned the position to give more time to his researches. In 1782 he won a prize offered by the Berlin Academy with an essay on the path of a projectile. In 1787 he was a member of a commission which connected the Royal Observatory in Greenwich with the Paris Observatory geodetically. Legendre later was a member of the commission which made up the metric system. In 1795, he was made professor at the Ecole Normale. Unlike Laplace, Legendre had little recognition. Laplace was always unfriendly to him and at length Legendre lost his pension as a member of the Académie and he died in poverty.

His greatest contributions to mathematics are contained in his *Mémoire sur les transcendantes elliptiques* (1794), *Essai sur le théorie des nombres* (1798), and his *Traité des fonctiones elliptiques et des intégrales eulériennes* (1827–1832).

What concerns us chiefly, however, is his *Eléments de géométrie*, published in Paris in 1794. This work went through many editions in France. Sir Thomas Heath notes that the edition of 1823 was the twelfth. A translation into English was made in 1819 by John Farrar who was then Hollis Professor at Harvard. The work was used at once in that university. Thomas Carlyle published another translation, anonymously, in Edinburgh in 1822. This was revised by James Ryan (1828) for the use of the Military Academy at West Point. Later, Charles Davies brought out the popular *Davies' Legendre* and still another revision was made by Professor Van Amringe of Columbia University. The text was the chief one used in this country prior to the publication of the first Wentworth *Geometry* in the seventies.

The text seems to have been written with the idea of making demonstrative geometry more rigorous than it had become at the

hands of the innovators of the seventeenth and eighteenth century many of whom had tried to make the subject "practical." The part of Euclid's *Elements* which dealt with geometry followed the sequence,—lines, areas, circles, regular figures, proportion, and proportion applied to plane figures. Legendre's scheme was,—lines, circles, proportion applied to plane figures, regular polygons and the circle, planes and dihedral angles, polyhedra, and the sphere. His treatment of solid geometry was much more extensive than Euclid's, and he permitted the use of lines and figures before the methods of constructing them had been demonstrated which Euclid had not allowed.

Sir Thomas Heath gives an extended account of the development of Legendre's ideas regarding parallel lines and of his attempt to prove the parallel line postulate. The theorems that entered into Legendre's final treatment were:

The sum of the three angles of a triangle cannot be greater than two right angles.

If the sum of the three angles of a triangle is less than two right angles, if a triangle is made up of two others, the deficit of the former is equal to the sum of the deficits of the other two.

If the sum of the three angles of a triangle is equal to two right angles, the same is true of all the triangles obtained by subdividing the triangle by straight lines drawn from a vertex to the opposite side.

If in a triangle, the sum of the three angles is two right angles, a quadrilateral can always be constructed with four right angles and four equal sides, exceeding in length any assigned rectilineal segment.

If the sum of the three angles of one triangle equals two right angles, the sum of the three angles of any other triangle equals two right angles.

If the sum of the three angles of any triangle is less than two right angles, then the sum of the three angles of any other triangle equals two right angles.

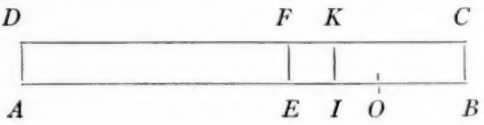
If the sum of the three angles of a triangle equals two right angles, through any point in a plane there can only be drawn one parallel to a given straight line.*

The reading of these theorems will effectively do away with the idea that Legendre made Euclid less rigorous.

Legendre's treatment of incommensurables is of considerable interest. He applies the method to each of the fundamental cases in the proportions between areas and lines, between lines and lines, and between the central angle and its intercepted arc. The simplest case, naturally, is the proof that two rectangles having the same altitude have areas that are proportional to their bases. Let us

* Heath, Thomas Little, *The Thirteen Books of Euclid's Elements*, 1926 ed., pp. 213–219.

suppose that the case has been proved for rectangles whose bases have a common unit of measure. Then if the rectangles $ABCD$ and



$AEFD$ have the same altitude AD, and if AB and AE have no common unit of measure, Legendre then assumes the ratio $\frac{ABCD}{AEFD}=\frac{AB}{AO}$ where AO is the missing term in the proportion.

Legendre then investigates the relation between AO and AE, considering first the case in which AO is greater than AE. AO is laid off on the base of the rectangle $ABCD$. Then AB is divided into segments of equal length, each being less than the segment EO. This means that at least one point of division will fall between E and O. Suppose that this point of division is I and draw IK perpendicular to AB. Then $ABCD$ and $AIKD$ have the same altitude and their bases are commensurable. Accordingly

$$\frac{ABCD}{AIKD}=\frac{AB}{AI}. \qquad \text{But} \quad \frac{ABCD}{AEFD}=\frac{AB}{AO}.$$

From these proportions, we have $\frac{AIKD}{AEFD}=\frac{AI}{AO}$. But, it is evident that $AIKD$ is greater than $AEFD$ while AI is less than AO, thus the assumption that AO is greater than AE leads to an absurdity. Similar reasoning shows that AO cannot be less than AE. Thus $AO=AE$ and $\frac{ABCD}{AEFD}=\frac{AB}{AE}$. This should be compared with Euclid's scheme which depended on his definition of proportion. Again, this instance would seem to indicate that Legendre's Geometry was no denatured version.

VERA SANFORD

Abstracts of Recent Articles on Mathematical Topics in Other Periodicals*

By NATHAN LAZAR
Alexander Hamilton High School, Brooklyn, New York

Algebra

1. Duncan, Dewey C. *Euler's Theorem: $E+2=F+V$. School Science and* Mathematics. 35: 40–3. January 1935.

After pointing out that the above theorem was known to Descartes, who flourished a century before Euler, the writer notes that the texts in Solid Geometry are almost unanimous in refraining from proving it. The few books that do attempt a proof go astray, because of inadequate analysis of all the assumptions that were made implicitly.

The idea of *connectivity of surfaces* is then introduced. "We say that a polyhedron (in fact, any surface whatever) *is simply connected* if it is completely divided into two pieces by any single closed intersection of the surface with a plane. Such a closed section of a polyhedron is evidently a plane polygon. The ordinary surfaces of elementary solid geometry are of this sort."

With the aid of this notion of connectivity of surfaces, which has important applications in collegiate mathematics, the author proceeds to "a very easy and adequate proof of the theorem."

2. Goodrich, Merton Taylor. *Algebra as a medium for the interpretation and control of nature. School Science and* Mathematics. 35: 16–7. January 1935.

A plea for teaching algebra as "a real, living, power-giving subject." This article is another in the great number of inspirational appeals, of which the pedagogic literature is full. It abounds in such expressions as "algebra is a subject shining with beautiful symbols," and "The study of algebra begins to give real pleasure just as soon as pupils see that it gives a broader and fuller meaning to life."

Granted that ideals and inspirations are necessary as motive powers for the propulsion of the human mind in any of its myriad endeavors, let us not forget that it cannot flourish on ideals alone. Just as the generation of steam is valueless until means are devised for its utilization in the actual pushing of pistons and turning of wheels, so is the spinning of fine phrases a mere exercise in imagination unless it is accompanied by explicit directions showing how to convert its implications into actual practices. For a while, at least, let us have less of vision and more of technique!

3. Lindquist, E. F. *The gap between promise and fulfilment in ninth-grade algebra.* The School Review. 42: 762–71. December 1934.

A report of the fifth annual Iowa Every Pupil achievement testing pro-

* Readers are requested to bring to the attention of the reviewer any pertinent article to which no reference has been made in this department within four months of its publication.—The Editor.

gram in elementary algebra which was administered to 9,034 ninth-grade pupils just completing a year's course in that subject in 230 Iowa High Schools.

"The algebra examination used in this program consists of sixty-two items organized into three parts, or sections. Part I contains twenty problems requiring, in most cases, only simple applications of fundamental processes. Part II contains thirty items testing the pupils' ability to express simple relations in algebraic form, or to interpret relations thus expressed, that is, testing his ability to use and to interpret the 'language' of algebra. Part III consists of twelve verbal problems of the kind typically presented in textbooks and in instruction."

The results were disheartening and discouraging and the following are some of the conclusions drawn from the data.

1. "A significant proportion of high-school pupils are, by reason of mental ability, previous training and present motivation, incapable of deriving enough of value from ninth-grade algebra as it is now generally taught to justify its being required of all pupils.

2. "The content and the methods of teaching high-school mathematics are seriously in need of reörganization and improvement." Without taking issue with the conclusions, the following remarks are pertinent.

   a. There is nowhere in the paper a description of the criteria used in making up the test.
   b. There is great doubt, ir the mind of one reader at least, as to the mathematical value of many of the questions that were put.
   c. Perhaps the results would not have been so disheartening if the recommendations to spread out the work of algebra over the seventh, eight, and ninth years were carried into practice. (see below item 5)

4. Read, Cecil B. *Relative efficiency of the two methods of approximating the roots of an algebraic equation.* School Science and Mathematics. 35: 30–4. January 1935.

The author describes an experiment to determine whether Horner's method or the graphic one is the more efficient in approximating the irrational roots of an algebraic equation of a degree higher than the second.

Because of the many difficulties due to the schedule of the classes, the author feels that the study is not conclusive, "but it does indicate very definitely the need for consideration of the possibility of a change. Additional experiments with larger number of pupils would be needed. No claim is made that the graphic method gives the theory of Horner's method. In fact, some of the topics necessary for the presentation of Horner's method may be omitted if attention is restricted to the graphic method. Perhaps in an elementary course we might restrict our work to the obtaining of a solution, leaving for a more advanced course more difficult or alternative methods.

"Accepting the more or less prevalent opinion that a correct answer is essential, the particular method being immaterial, the results indicate, (a) That a solution by successive graphs enables the student to obtain the desired results with less expenditure of time than Horner's method. (b) Moreover, it would seem that less time is required for the presentation of the subject matter to the students. With these advantages, teachers might well consider the possibility of a change in the method used in handling this particular part of algebra."

5. Reeve, W. D. *What is the solution?* National Mathematics Magazine. 9: 146–7. February 1935.

A rejoinder to an article by Chas. H.

Sisam of Colorado College that appeared in that magazine in December 1934 under the title "A Startling Paradox." In it it was recommended that the mathematics of the elementary and secondary schools be moved bodily, a year or two, later in the curriculum, because mathematics is so hard that pupils do not get as much from its study as they should.

"Is mathematics hard for pupils because they study it at too early an age, or is it hard because they do not start to study it early enough? I believe that the situation would be greatly improved if, instead of trying to crowd elementary algebra all into the ninth grade, we should begin the study of the subject in the seventh grade and then spread it out through the eighth and ninth grades. Similarly, if instead of crowding all of the difficulties of geometry into the tenth grade, we should begin the study of informal geometry in the seventh grade and then develop the subject continuously through the tenth year, we should diminish the pupils' difficulty considerably.

"A continuous course in general mathematics beginning in the seventh grade with the emphasis upon mastery at the various levels, and the development of fundamental concepts as the main aims of such a course might help to bring about a renaissance.

"In the light of modern thought some of the traditional and now obsolete topics, which we have inflicted upon the pupils, are not suitable for teaching at any age. Moreover it is assumed that no matter when a subject is introduced, a good teacher is indispensable."

Arithmetic

1. Acker, Lela. *Making arithmetic attractive.* The Volta Review. 36: 709–11+. December 1934.

Although this is a description of the devices and games the author found helpful in "selling" arithmetic to deaf children, they can be also used, to good advantage in teaching normal children.

2. Brown, George A. *Units of study in relation to use and understanding.* Journal of Educational Research. 28: 237.

A description of a method of teaching, through the medium of stories, the "use of zero so that number operations can be carried to any number of places."

3. Crocker, Gladys Hosmer. *Number stories useful in the teaching of simple number facts.* Grade Teacher. 52: 40+. January 1935.

Stories, pictures and explicit directions on the successful methods of enticing a child into learning "How many pints in a quart" and "The months of the year."

4. Greenwood, Ella. *Problems in junior arithmetic.* School. 23: 211. November 1934.

The writer shows how the fundamental facts of arithmetic can be taught by playing "grocery store" or "drug store." The desire to be "hired" as a clerk served, for many, as incentive to become more interested in the study of arithmetic and more proficient in the calculations.

5. Lockhead, Jewell. *The Play way: material to stimulate interest in reading and numbers.* The Grade Teacher. 52: 30+. January 1935.

The author describes the various uses to which certain posters, cards, charts, toys and puzzles can be put in stimulating an interest in reading and numbers. The material "meets three requirements:

a. It appeals to the play interest of the child.
b. It conveys definite ideas.
c. It is so graded as to provide experi-

ences in each of the early stages of reading and number work."

6. Moe, George A. *How to correlate mathematics and history.* Minnesota Journal of Education. 15: 144–45. December 1934.

The author does not *merely* wax enthusiastic over the beauty, the importance and the utility of correlating different subjects in the same curriculum. He gives, instead, *eight specific, concrete situations and illustrations* of the possible correlation between eighth grade arithmetic and eighth grade history.

7. Myers, Garry Cleveland. *Hard Subtraction: how to lay a good foundation for work in arithmetic.* Grade Teacher. 52: 41+. December 1934.

An analysis of the troubles pupils find with subtraction and a program for remedying the difficulties.

8. Myers, Garry Cleveland. *Percentage: teaching methods based on every day interests.* Grade Teacher. 52: 40+. January 1935.

A detailed presentation of a method of teaching percentage to intermediate and grammar grades. The author includes blackboard exercises, useful explanations, typical verbal problems, and other relevant, useful hints.

9. Rich, Frank M. *Project arithmetic: numeration to 100 and the tables.* American Childhood. 20: 9–10. January 1935.

Showing how games can be used to give the children "a new deal in arithmetic with such radical changes in material and methods as will make the subject more comprehensible to average and subaverage minds, and more in accord with the healthy, live, social interests of childhood."

10. Wentland, C. R. *Visualizing percentage problems.* Wisconsin Journal of Education. 67: 173–4. December 1934.

A description of a method, mainly pictorial, whereby to conquer the bugaboo of the sixth and seventh grades. The kind of mathematical thinking the author would like to inculcate cannot be adequately summarized, but can best be represented by a quotation.

"In the problem—'Find the number of which 16 is 5%'—we understand that the lazy number does no work, but on the contrary it may hinder work by standing in the way. Such 'loafers' [the numbers, not the pupils! N. L.] must be dealt with in a special way, that is by putting them in jail, the jail being the division bracket. Then we have $.05\overline{)16}$ in which the 5% (rate) works by standing guard until the base is found. We establish a similar situation in the problem—'6 is what percent of 90?'—where the 90 stands guard. Pupils readily [sic!] grasp that with a worker in jail and a lazy guard the jail will be wrecked and nothing is accomplished to help in the finding of the third number."

Comments are unnecessary.

Geometry

1. d'Ocagne, M. *Étude rationelle du problème de la trisection de l'angle.* (*A logical study of the problem of trisecting an angle.*) L'Enseignment Mathématique. 33: 49–63. (1934.)

The author describes three methods of trisection, which are not, of course, theoretically accurate, but which attain a high degree of approximation. The interesting feature of this study is that the *theoretical* degree of error in each procedure is calculated and is found to be astonishingly low.

2. Henry, Lyle K. *The rôle of insight in plane geometry.* Journal of Educa-

tional Psychology. 25: 598–610. November 1934.

The paper presents aspects of the results of an experiment performed at the University of Iowa, under the direction of Dr. F. B. Knight.

"The purpose of the experiment was to test the hypothesis that the mental behavior observed in solving originals in geometry under controlled conditions could be adequately and correctly described as the operation of 'insight.'

"In order to test this hypothesis it was necessary to do three things:

a. Determine, in so far as possible, adequate and satisfactory criteria of insight.
b. Set up a controlled situation in geometry which offered opportunity for the characteristics of insight to appear.
c. Observe whether or not the characteristics of insight appeared universally, frequently, rarely or not at all."

The author gives a careful analysis of the notion of "insight" and then proceeds to describe in great detail the apparatus, the technique and the results. A bibliography of 28 items is also included.

It would be futile to attempt an intelligible summary, with the space at our disposal, of an article of 13 pages which is itself a résumé of a much bigger work. But the concluding remark of the writer must be quoted. "While insight occurred in the solution of originals, it was not the robust, universal trait that characterizes the rational solution of problems, and was, therefore, inadequate and unsuited to describe the typical behavior in this experiment."

It is regrettable that a thesis dealing with a problem of utmost importance to teachers of geometry should be reported *exclusively* in the Journal of Educational Psychology. Could not the National Council of Teachers of Mathematics do something about creating one central clearing-house for all Masters' and Ph.D. theses dealing with mathematical pedagogy?

3. Hoadley, Edwin A. *Live and learn.* Education. 55: 202–05. December 1934.

A few gentle hints, not entirely new, for the teacher of geometry or any other subject. "First stimulate interest; then reveal difficulties, encourage discovery, hint at mysteries to be solved. Watch for growth and praise the first green sprouts. Later one of the plants will have enough energy within itself to grow without much attention."

4. Smith, David Eugene. *Euclid, Omar Khayyâm, and Saccheri.* Scripta Mathematica. 3: 5–10. January 1935.

A report on a mathematical manuscript discovered in a Persian library in 1933.

The value of the manuscript is threefold:

a. It is a copy of a commentary by Nasîr ed-dîn, who flourished in the years 1201–1274 A.D.
b. It contains an important portion of a mathematical essay by the Persian poet and mathematician, Omar Khayyâm (c. 1044–1123 A.D.), which appears to be the first chapter of his Commentary on the Difficulties of Euclid.
c. But the most important phase of the discovery of the manuscript is the following:

1. Girolamo Saccheri (1667–1733), whose *Euclides ab omni naevo vindicatus* (Euclid vindicated from very flaw) is generally considered as the first step in a non-Euclidean geometry, used the same lemma as the one of Nasîr ed-dîn, which, the author shows is due to Omar Khayyâm.

2. Saccheri was familiar with the work of John Wallis who mentions Nasîr ed-dîn in his works.
3. Many of Omar Khayyâm's suggested theorems are essentially the same as the first few of Saccheri's; the proofs are also the same and in some cases identical.
4. It seems, then, that Omar Khayyâm is to be credited with being the first to get an inkling of the *possibility* of a non-Euclidean geometry, although Saccheri still remains the first to have *actually* hinted at the new type of geometry.

Miscellaneous

1. Bailey, Ralph G. *The effect on the achievement in physics, of drills on mathematical skills needed in high school physics*. School Science and Mathematics. 35: 89–91. January 1935.

"Just what mathematics is needed in physics has been analyzed and reported in several studies, the most important of which is that of Kilzer in 1928. From the results of these studies it was possible to build a test which would check each pupil on each type of skill and to construct a set of drills for the development of mastery of the required skills as indicated by the test." The article is the report of an attempt to check the value of such a remedial program. The conclusions are:

a. The drill on the mathematical skills did definitely improve the mathematical ability of the experimental group.
b. The drill on the *mathematical skill* improved the ability of the experimental group in solving typical *problems of physics*.
c. Although part of the class time was used for drill on mathematical skills, the experimental group was, nevertheless, superior in achievement in physics.

2. Dresher, Richard. *Training in mathematical vocabulary*. Educational Research Bulletin. 13: 201–04. November 1934.

The purpose of the study was to determine the effects of specific training in the technical terms of the subjects on the mathematical vocabulary of the student as well as on his ability to understand and solve problems.

"Approximately five hundred pupils participated in the experiment, 250 each in the experimental and control groups. They were all given the same tests at the beginning of the second semester. The experimental group was given special training in vocabulary; there was special attention given to the words as they appeared in the book and a list of technical words and their definitions was mimeographed and given to each pupil in the experimental groups. Tests on vocabulary were given at various times during the semester to motivate the learning of these technical words. At the end of the semester the same tests were given to all the pupils."

The results of the experiment are:

a. Under the usual teaching conditions there was comparatively little advance in knowledge of mathematical vocabulary.
b. As a result of specific training in mathematical vocabulary there was not only a definite and consistent gain in knowledge of technical vocabulary (which might have been expected) but *an increase in the ability to solve problems as well*. These results were obtained despite the fact that the experimental groups were poorer human material than the control group.

The implications for the teacher of

mathematics are too obvious to warrant any comment.

3. Ginnings, P. M. *Mathematics and science requirements for the liberal arts degree in southern colleges.* High School Quarterly. 23: 10–12. October 1934.

A study of the mathematics requirements in 102 colleges gave the following results: mean, 4.1 semester hours; median 6.0; fifty percent of the colleges required six semester hours and thirty-five percent required no mathematics.

Other conclusions follow:

a. The size of the educational institution has no significant effect on the minimum mathematical and scientific requirements.
b. Results are inconclusive on the question whether some states were outstanding in this respect.
c. Comparison of requirements of southern colleges with a random sampling of the north-central association group shows tentatively that the requirements are lower in the northern than in the southern colleges. "Apparently this is due to a broader and more extensive use of the elective system and not to a lower evaluation of mathematics and science in the educational curriculum."
d. A comparison of the means and medians of the required semester hours indicates that the mathematical and scientific requirements in the endowed and denominational educational institutions of the south is larger than those in the state institutions.

4. Moore, Charles N. *Mathematics and Science.* Science. 81: 27–32. January 11, 1935.

The address of the vice president, and chairman of the section of Mathematics, American Association for the Advancement of Science, Pittsburgh, December 1934. It is a thoroughgoing analysis of the rôle that mathematics has played in the history of scientific endeavor and of the mutually beneficial and stimulating influences that mathematics and the experimental sciences have exerted upon each other.

In these times, when mathematics is under suspicion and attack, it is good for one's soul and intellectual integrity to come across such an article. It should be encouraged as voluntary reading for the friends and teachers of mathematics, and should be made compulsory reading for its critics and enemies.

The editor of this Journal deems it of such importance that he plans either to reprint it in its entirety in an early issue of "The Mathematics Teacher" or to have it published privately for free distribution among educators and laymen.

NEW BOOKS

Professional Treatment of Subject Matter of Arithmetic, for Teacher Training Institutions, Grades I–VI. By Elias A. Bond, Bureau of Publications, Teacher's College, Columbia University.

In 1920, Professor Bagley made his plea for the professional treatment of subject matter in teacher training institutions. Following this, Dr. Randolph made extensive studies along this line; in 1925 and 1926, Professor Upton, in a series of articles entitled "Studies in the Teaching of Arithmetic" in the Teacher's College Record, showed how the professional treatment of subject matter should be carried out in arithmetic. It was the hope of a great many teachers of mathematics that these articles would be continued or expanded into a text.

Dr. Bond has now rendered this service. He is a highly successful, experienced teacher who has devoted a lifetime to the teaching of mathematics and teacher training. He is particularly interested in the teaching of arithmetic, and has produced an outstanding text which for the first time gives a full and successful professional treatment of the subject matter of arithmetic.

The historical background together with the necessary psychological and philosophical phases of each topic prepares the reader to appreciate and understand the methods suggested. Where an algebraic treatment is necessary to give a full generalization or proof, Dr. Bond does not hesitate to use algebra. This, however, may be easily understood and followed by any person who has had a first course in algebra. The text is sufficiently scholarly to command the interest and respect of the best students that we are now getting in our normal schools and teachers' colleges.

At the end of each chapter, the author gives a very full list of exercises that thoroughly test the readers' ability to comprehend the material given and to apply the principles developed to concrete situations. Following the exercises is a long list of "references suggested for required reading," and at the end of the book there is a very good chapter on the "Professional Treatment of Testing in Arithmetic" and one on "Organization of Teacher Training Courses in Mathematics."

Dr. Bond reviews and evaluates all the worth while studies in arithmetic and is very thorough and fair in the treatment of the various controversial issues in the subject.

Dr. Bond's book is unquestionably the best modern text on arithmetic methods for teacher training institutions. It should be read by all teachers and supervisors of arithmetic and even by high school and college teachers, who will find it interesting and profitable reading. The author should continue the good work done in this text by writing another volume to cover the very rich filed of seventh and eighth grade mathematics.

C. N. Shuster

Integrated Mathematics with Special Application to Plane Geometry. By John A. Swenson. Edwards Bros., Ann Arbor, Michigan, 1934.

In both subject matter and method of presentation this book is a unique contribution to tenth year mathematics and is outstanding among text books published in America during the past year.

Throughout the book, Dr. Swenson leads up to new ideas gradually with exercises that are carefully selected and uniformly well graded. Before introducing formal theorem proofs, he gives the student by means of exercises the concept of what a proof is, as well as the method of constructing a proof. He stresses geometry as a method of thinking. He shows in the subject matter how the "If-then-Kind" of thinking referred to in the introduction becomes more "critical and artistic" as one uses it more and more.

The text is very usable because the exercises are so carefully related to the theorems. The difficulties are brought in slowly enough for the student to master each one. The book is exceedingly well planned, so that it is best to follow the work as the author presents it in the text. A teacher can assign a lesson and know that the class may study the text for advanced work without meeting an insurmountable difficulty, even though the advanced work has not been fully discussed in class. Actual experience shows that the students like the concise wording of theorems, axioms, and other statements. These are both brief and meaningful.

Among the unique features of this book that are especially worthwhile is the treatment of congruency by a graphic study of relations of sides and angles of a triangle. This is an expansion of the article by Dr. Swenson in the Fifth Year Book of the National Council of Mathematics Teachers, stressing agreement and dependence in such a vigorous and interesting way that the student can not fail to get the relationships. This functional treatment is stressed repeatedly in connection with angle measurements, tangents and secants to a circle from an external point, areas, and similarity. In connection with similarity, a new notation has been introduced which greatly simplifies the treatment of the entire topic.

As a result of years of interested study of European texts, and experimenting with teaching the beginnings of analytics and calculus in his high school classes, Dr. Swenson has put in this book on Integrated Mathematics, the fundamental ideas of motion, coordinate geometry, slope, area, volume, so naturally and easily that he gives the student not only the elements of geometry as a mode of thinking but also a splendid foundation for work in algebra, trigonometry, analytics, and calculus.

In the chapter on third and fourth dimensions even the slow student gets the idea of the fourth dimension while the alert student is eager to make further study of it. The last section is given to problems involving applications of mathematical reasoning in everyday life.

No ideas new to the student in tenth year mathematics are forced on him without understanding. Instead, the text shows the work of a genius, a genuine mathematician, who knows the subject matter and can present it on the level of the high school student, leading him on in a most interesting and stimulating way. The book abounds in suggestions for both teacher and student.

M. Bird Weimar

Plane and Spherical Trigonometry. By Claude I. Palmer and Charles W. Leigh. McGraw-Hill Book Company, Inc., 1934, XIV+229.

This book is a revision of an earlier edition. This new edition presents a new set of problems where the change consists in details rather than a change in type.

The book is intended to serve primarily those students who are in schools of technology, but the authors are hope-

ful that teachers of mathematics in classical colleges and universities may find the text suited to their needs.

Teachers who have used the older book with success will certainly find the new edition acceptable and others will be interested to see how the subject is herein developed.

Plane Trigonometry (and Four-Place Tables of Logarithms). By W. A. Granville, P. F. Smith, and J. S. Mikesh. Ginn and Company, 1934, x+212. Price, $1.60.

This book is a revision of an earlier and very widely used edition of Granville's "Plane Trigonometry." The authors have tried to keep intact the features of the older text which have met with general approval. Wherever changes have been made they have been made for the purpose of change in emphasis. The treatment of trigonometric identities and equations has been revised so that now the applications can be completed before trigonometric analysis is taken up.

Four-place tables are given as in the original edition with the addition of tables giving the rational values of the sine and cosine, tangent and cotangent, and some explanation on the use of the tables.

A Brief Course in College Algebra. By Walter Burton Ford. The Macmillan Company, 1933, VII+288. Price, $2.25.

This book is a revised edition of the author's earlier work. It contains a chapter on complex numbers not given in the earlier book for those courses where such work is needed and also a collection of 167 miscellaneous exercises from elementary algebra for those teachers who wish to review that field.

The usual topics considered necessary in college algebra are included as in the earlier edition and there is ample material to enable any student who is serious about his work to obtain a good background in the subject.

A Short Course in Trigonometry. By James G. Hardy. The Macmillan Company, 1932, IX+181 pages with tables. Price, $2.25.

This book is the result of using the same material in lithotyped form for three years with several sections of students. It is written for students of some maturity who really want to know the nature and value of trigonometry.

The material is presented in such a way that continuity and correlation with other fields is a feature while at the same time the course is not too long as many of our traditional courses have been. For present needs it is clear that we can well afford to cut down the obsolete features of trigonometry and concentrate on what is fundamental as the author has tried to do in this book.

Elements of Analytic Geometry. By Clyde E. Love. The Macmillan Company, 1932, XI+145. Price, $1.60.

This new edition of the authors "Analytic Geometry" is an abridgment rather than a revision of the older book. It is intended for use in courses where time does not permit such a large treatment of the subject. In any case the idea seems sound and commendable as it is now clear that not every student of mathematics needs to do all of the topics and problems of our earlier traditional courses in analytic geometry.

Teachers who have been looking for a shorter treatment of the subject which still maintains the essential material for clear understanding of the subject will be interested in this book.